Regression Error

Grinnell College

May 3, 2024

Today

- Regression posits linear relationship between dependent variable y and independent variable X of the form

$$y = \beta_0 + \beta_1 X + \epsilon$$

- Expand this to include combinations of independent variables
- We will talk about the error term on Friday

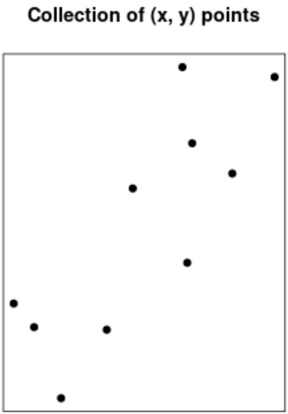
Error Terms

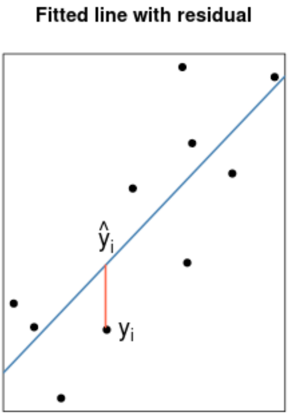
$$y = \beta_0 + X\beta_1 + \epsilon$$

Assumptions:

- Linear relationship between X and y
- Error term is normally distributed, $\epsilon \sim N(0, \sigma)$
- Error should be the same for all values of X, i.e., error same for all observations

Analyzing the error terms gives us a way to test the assumptions of our model


Collection of (x, y) points

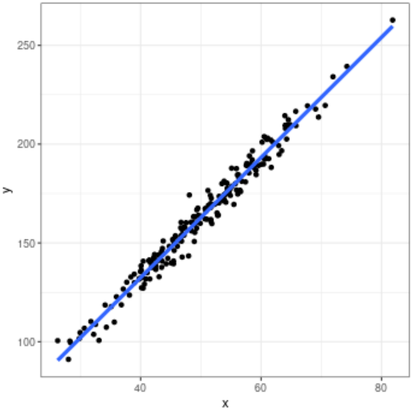

Fitted line with residual
$\hat{y}_i$
y_i

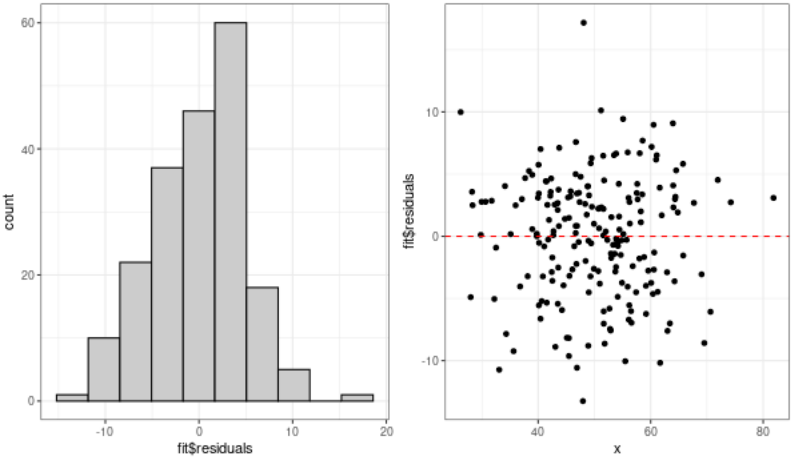
Part 1: Checking Assumptions

Residuals and assumptions

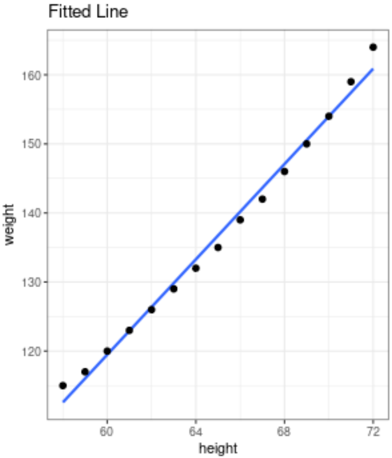
Three common ways to investigate residuals visually:

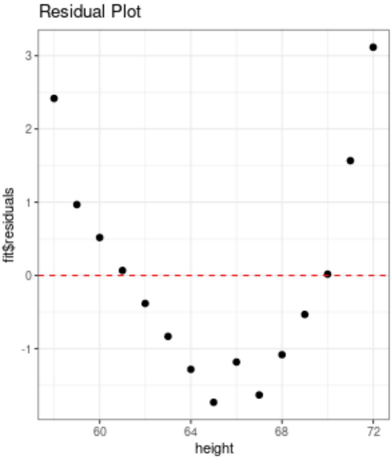
1. Plot histogram of residuals (normality)
2. Plot residuals against covariate (linear trend, homoscedasticity)
3. Plot residuals against new covariates (pattern identification)




count
fit$residuals
fit$residuals
x
60
40
20
0
-10
0
10
20
10
0
-10
40
60
80

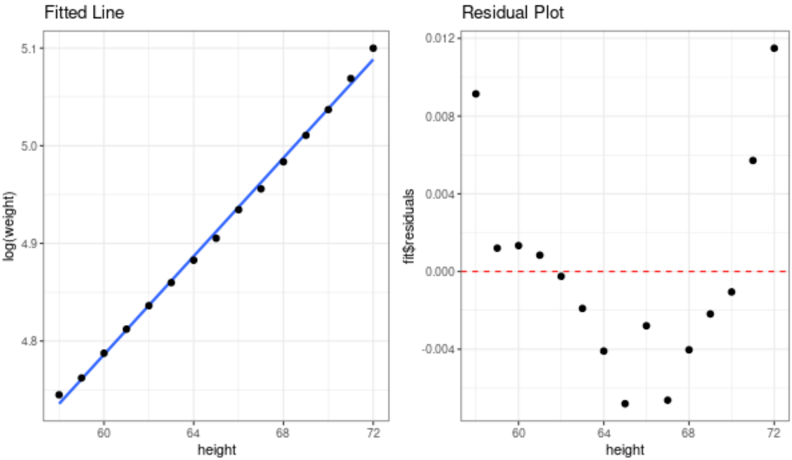
Tests of linearity





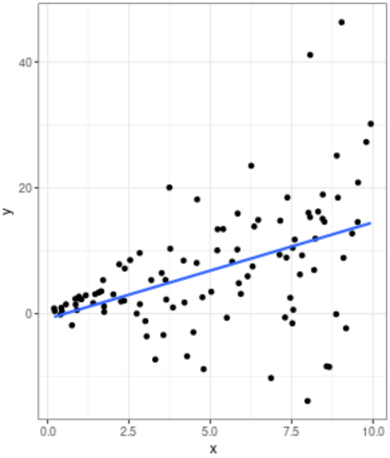
Tests of linearity

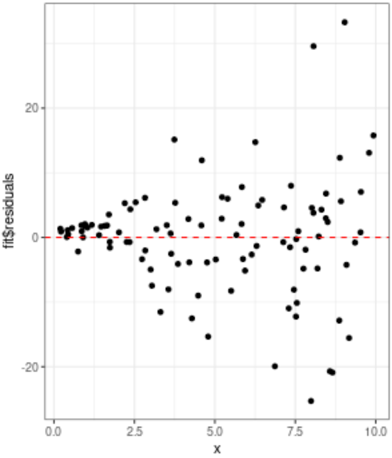
Sometimes a transformation of a variable (in this case, log(weight)) can help correct trends



Heteroscedasticity

Hetero = different, scedastic = random



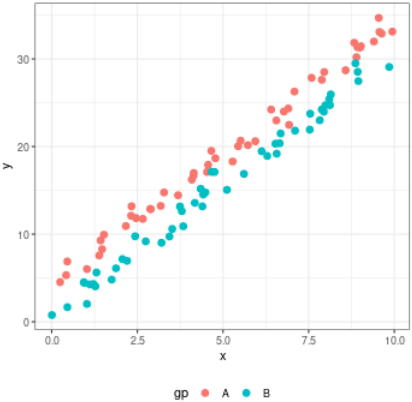


Part 2: Investigating Patterns

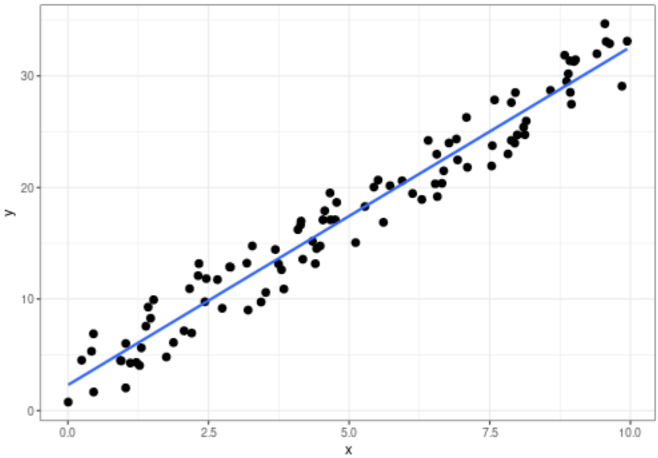
Considering new covariates

Suppose I have:

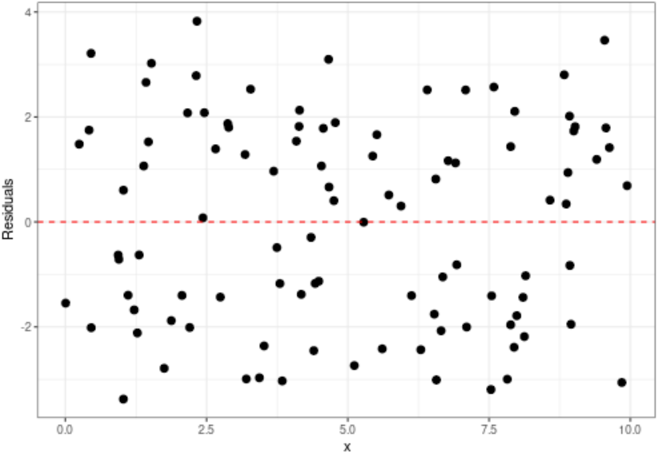
- Quantitative outcome y
- Quantitative predictor X
- Categorical predictor gp



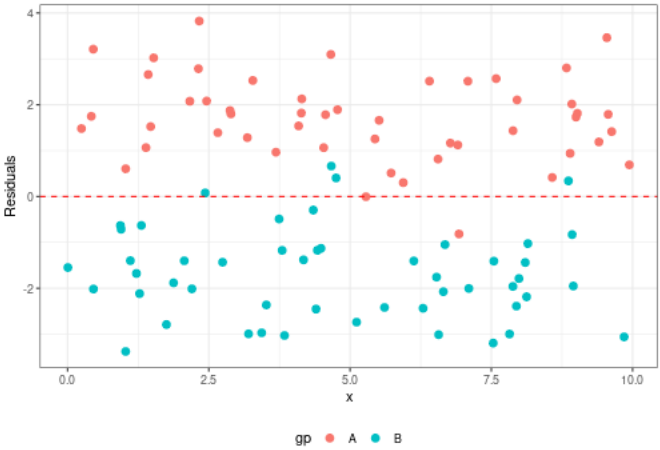
Considering new covariates



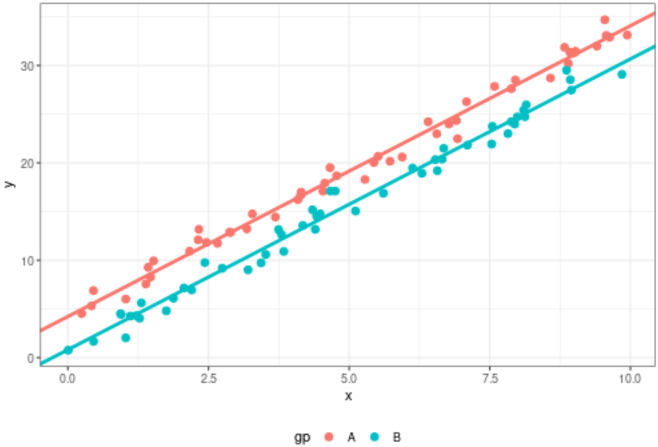
Considering new covariates



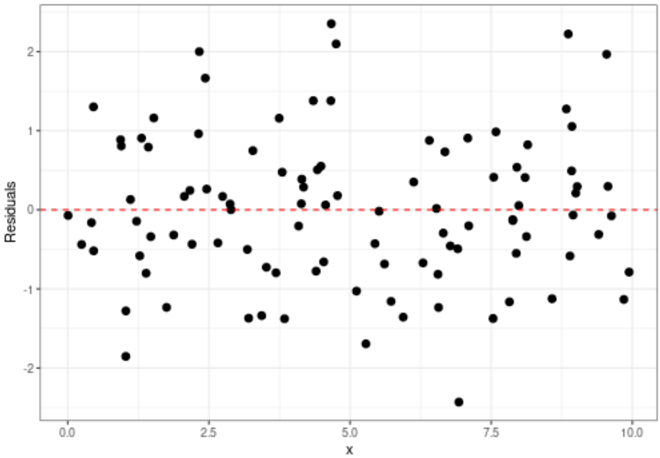
Considering new covariates



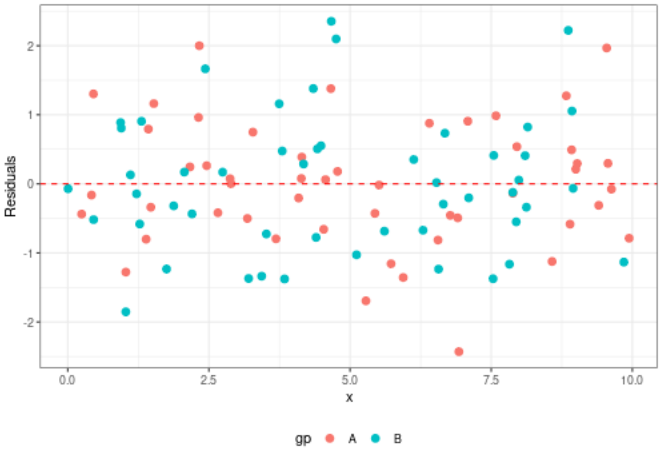
Considering new covariates



Considering new covariates



Considering new covariates



Correlated Covariates

Consider a simple linear model in which a covariate X is used to predict some value y

$$\hat{y} = \hat{\beta}_0 + X\hat{\beta}_1$$

The residuals associated with this describe the amount of variability that *is yet to be explained*

$$r = \hat{y} - y$$

The idea is to find new covariates *associated* with this residual, in effect "mopping up" the remaining uncertainty

Considering new covariates

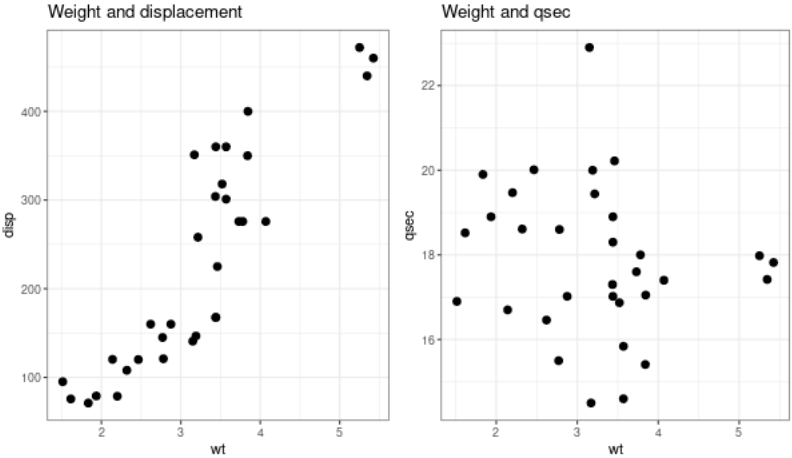
On Wednesday we considered an example predicting vehicle fuel economy with three separate models:

1. Using weight
2. Using weight and engine displacement
3. Using weight and quarter mile time

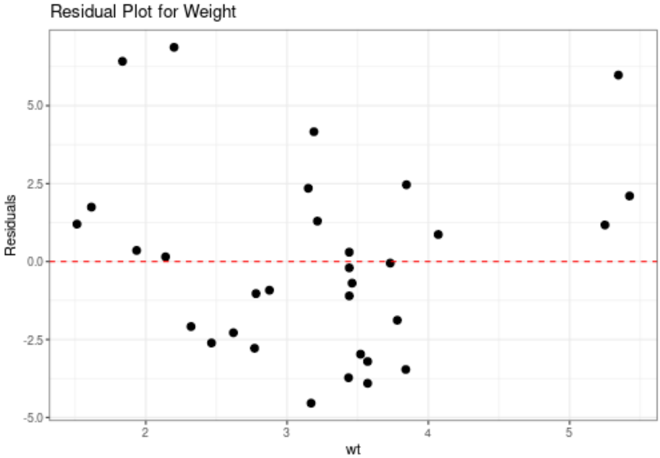
Correlated Covariates

```
> lm(mpg ~ wt, mtcars) %>% summary()

              Estimate Std. Error t value     Pr(>|t|)
(Intercept)   37.285      1.878   19.86   < 0.000002 ***
wt            -5.344      0.559   -9.56     0.000013 ***
R-squared = 0.75

> lm(mpg ~ wt + disp, mtcars) %>% summary()

              Estimate Std. Error t value      Pr(>|t|)
(Intercept) 34.96055    2.16454   16.15  0.000000049 ***
wt          -3.35083    1.16413   -2.8        0.0074 **
disp        -0.01772    0.00919   -1.93       0.0636 .
R-squared = 0.78

> lm(mpg ~ wt + qsec, mtcars) %>% summary()

              Estimate Std. Error t value       Pr(>|t|)
(Intercept)   19.746      5.252    3.76        0.00077 ***
wt            -5.048      0.484  -10.43 0.000000000025 ***
qsec           0.929      0.265    3.51        0.00150 **
R-squared = 0.82
```

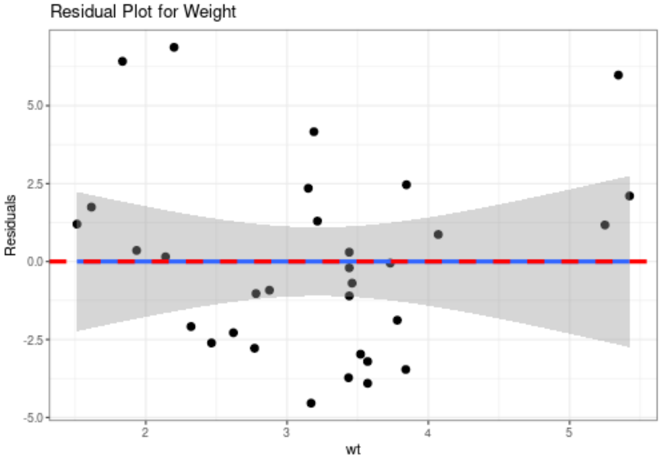
Correlated Covariates



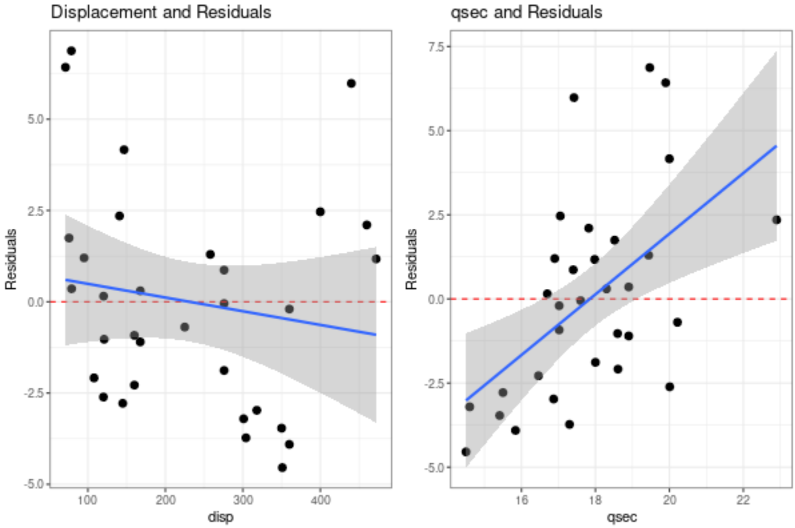
Residual Plots



Residual Plots



Residual Plots



Key Takeaways

1. Number of assumptions for linear model
   - Linearity
   - Normal errors
   - Homoscedasticity
2. Need way to determine which new variables to add to model
3. Examining errors effective way to test assumptions and investigate new covariates