Odds and Odds Ratios

Grinnell College

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Making Tables

A coin was flipped twice in succession and the face was recorded. This experiment was repeated $n = 10$ times

- Use the results to construct a two-way table or summary
- What is the marginal distribution of the first flip?
- What is the conditional distribution of the second flip given that the first was heads?

First	Second
T	T
H	H
H	H
T	T
T	T
T	H
H	T
H	T
H	H
H	T

Sorted Tables

First	Second
T	T
H	H
H	H
T	T
T	T
T	H
H	T
H	T
H	H
H	T

$\Longleftrightarrow$

First	Second
H	H
H	H
H	H
H	T
H	T
H	T
T	H
T	T
T	T
T	T

What we do today

- Introduce odds and probabilities
- Contingency tables
- Odds ratios

Odds and Probability

When dealing with a *binary* event, we often speak in terms of **odds**, a *ratio* of "number of successes" to "number of failures"

$$\#\text{ success} : \#\text{ failure}$$

This is distinct from the idea of **probabilities**, which give a ratio of the "number of successes" to the number of possible outcomes

$$\begin{aligned}\#\text{ success} &: \#\text{ total outcomes} \\ &: \#\text{ success} + \#\text{ failure}\end{aligned}$$

Odds

Suppose we have a 6-sided die, and we are interested in rolls that land on either 1 or 2.

$$\text{Die} = \{1, 2, 3, 4, 5, 6\}$$

- The *probability* of rolling a 1 or 2 is $1/3$
  1. There are 6 possible outcomes
  2. There are 2 possible successes
  3. Probability is $2 \ / \ 6 = 1/3$
- The *odds* of rolling a 1 or 2 are 2:4 (or 1:2)
  1. There are 2 possible successes
  2. There are 4 possible failures
  3. The odds of success are 2:4 (or 1:2)

Contingency Tables

A **contingency table** is a special two-way table in which both categorical variables have a binary response

	Event	Non-Event
Exposure	A	B
No Exposure	C	D

Specifically, we have event and non-event (order matters)

Determining Association

Situation 1:

	Event	Non-Event
Exposure	6	2
No Exposure	3	2

Situation 2:

	Event	Non-Event
Exposure	103	2
No Exposure	100	2

1. Difference in odds for each situation?
2. Ratio of odds for each situation?

Odds Ratio

An **odds ratio** is the ratio of odds between two groups

	Event	Non-Event
Exposure	A	B
No Exposure	C	D

- The odds of an event for the exposure group are A:B (or A/B)
- The odds of an event for the no exposure group are C:D (or C/D)

The **odds ratio** for these groups is then the ratio of their odds:

$$\theta = OR = \frac{A:B}{C:D} = \frac{A/B}{C/D} = \frac{A \times D}{B \times C}$$

Group Rows

Changing the rows will change which group is being compared to which

Case 1:

	Enjoy Ice Cream	
Age	Yes	No
Child	16	4
Adult	4	8

Case 2:

	Enjoy Ice Cream	
Age	Yes	No
Adult	4	8
Child	16	4

Event vs Non-Event

Which column is our "Event" also changes how we report our results

Case 1:

	Enjoy Ice Cream	
Age	Yes	No
Child	16	4
Adult	4	8

Case 2:

	Enjoy Ice Cream	
Age	No	Yes
Child	4	16
Adult	8	4

Considerations

From these examples, there are a few things that are worth considering:

- For a given odds ratio θ, the order of columns or rows will give us either θ or $1/\theta$
- If we compute a value $\theta < 1$, "flipping" the statement will allow us to report an odds ratio greater than 1
- What inferences can be made if we find that $\theta = 1$?
- The easiest thing you can do is make odds seem difficult
  1. Think about what you want to say
  2. Find the odds for each group
  3. Report the ratio

Example

Here, we tabulate survivors from the Titanic based on age

	Survived	
Age	Yes	No
Child	57	52
Adult	654	1438

Produce three valid statements regarding odds that can be made from this table (e.g., "The odds of an *adult surviving* are ... times the odds of a *child surviving*")

Odds Ratio Summary

- Odds and probabilities
- Column/row order matters
- Odds ratios
- $OR > 1$, $OR = 1$, $OR < 1$
- Always ask yourself: "Does what I'm saying make sense?"