Likelihood

Grinnell College

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Warm-up

Suppose I am interested in determining the proportion of countries in which the average life expectancy is greater than 70 years old. To this end, we collect sample of 25 countries, finding that 13 of them have life expectancy greater than 70

- What is the estimated proportion of countries with LE > 70?
- How would you construct a 95% confidence interval for the true value of π?
- How would you test the hypothesis $H_0 : \pi_0 = 0.6$?

Inverting Score

Let

$$z = \frac{\hat{\pi} - \pi_0}{\sqrt{\pi_0(1-\pi_0)/n}}$$

Then for given $\hat{\pi}$ and n, our 95% CI are all of the values π_0 such that

$$|z| < 1.96$$

Inverting Score

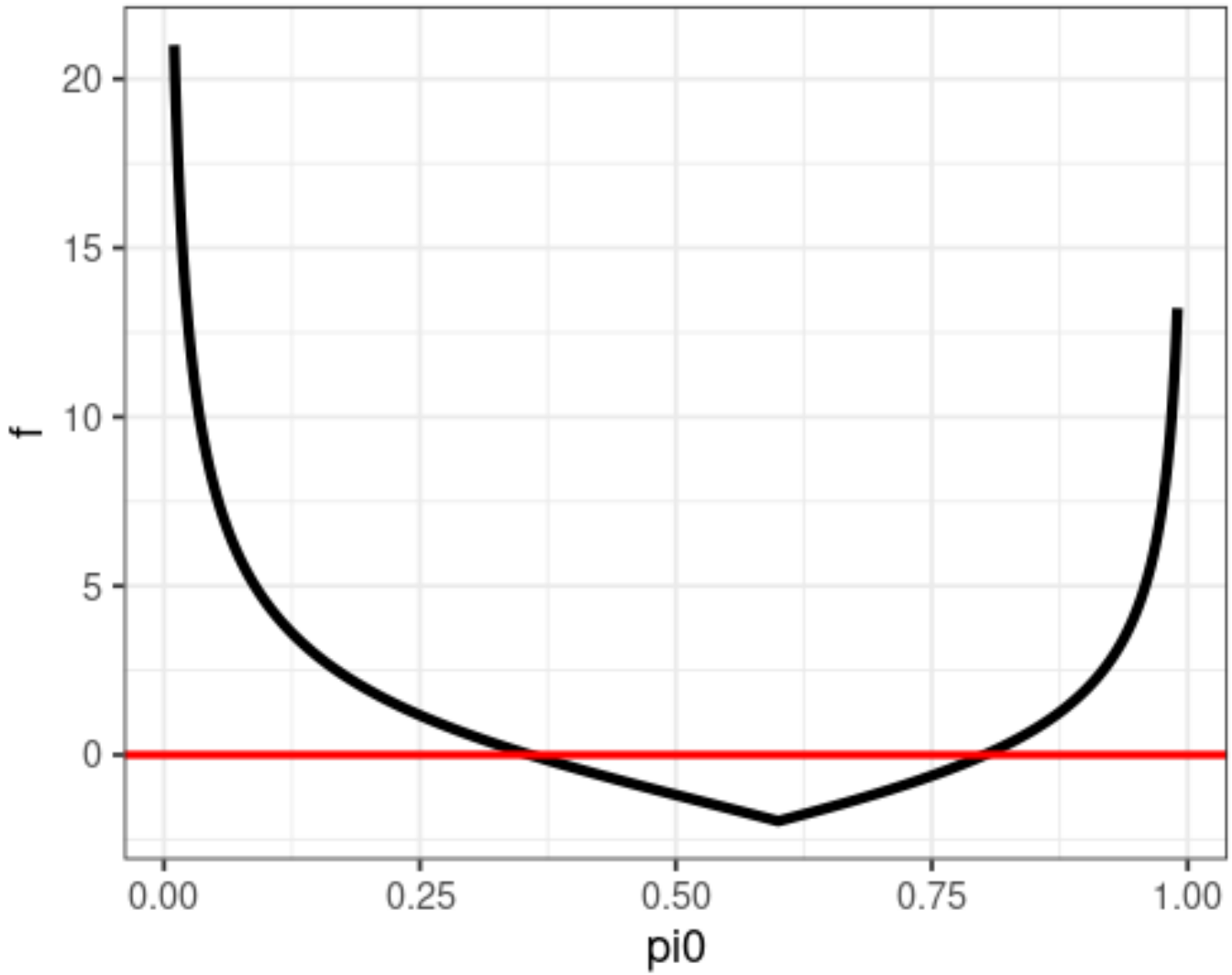
We can express this idea as:

$$f(\pi_0 \mid \hat{\pi}, n) = \left| \frac{\hat{\pi} - \pi_0}{\sqrt{\pi_0(1-\pi_0)/n}} \right| - 1.96$$

Where the values of π_0 such that $f \leq 0$ will make up our confidence interval

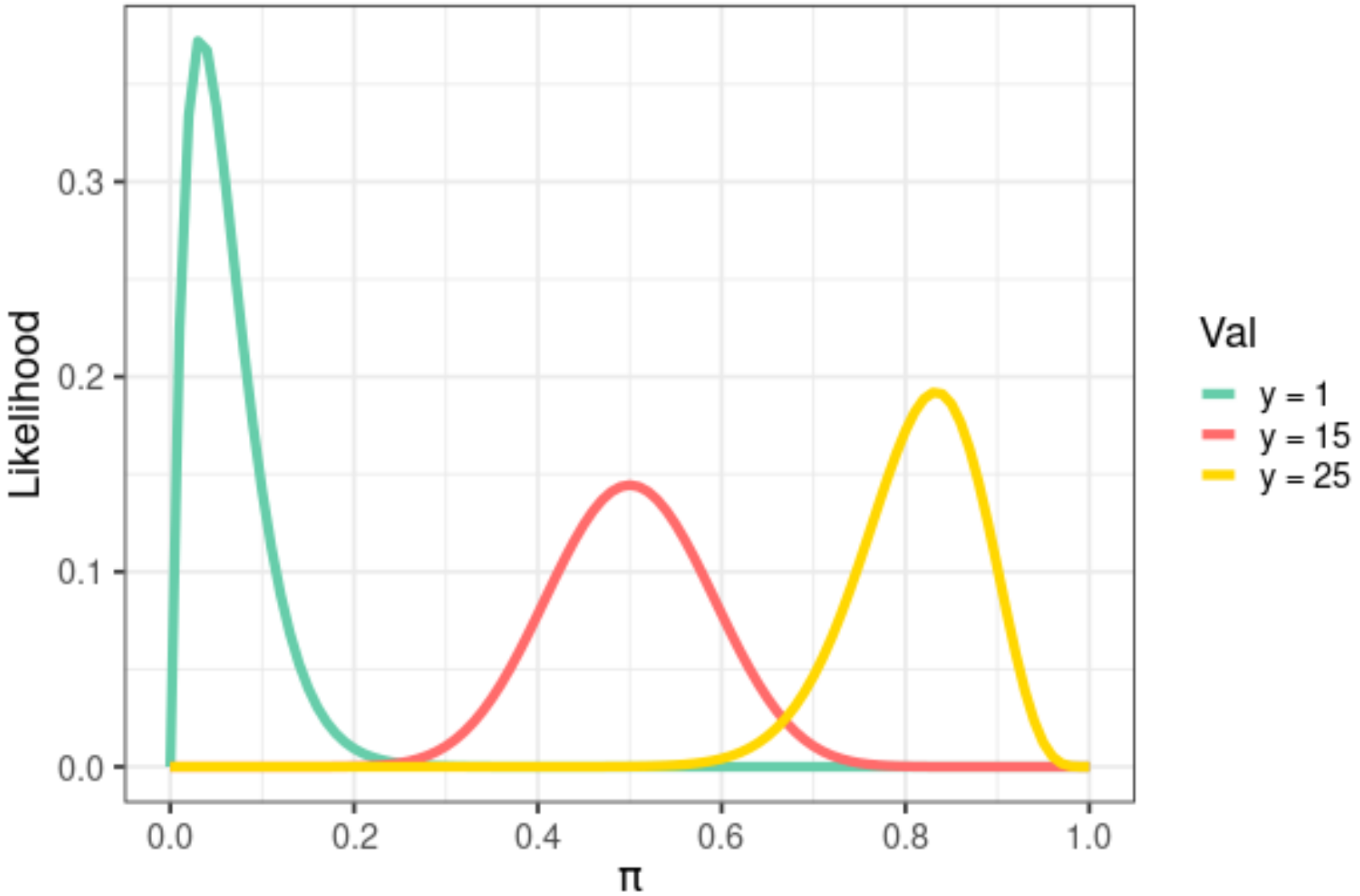
Inverting Score

Suppose $\hat{\pi} = 0.6$ and $n = 15$. We find the zeros of this function to be approximately (0.35, 0.80). Compare this with a Wald CI of (0.35, 0.85)



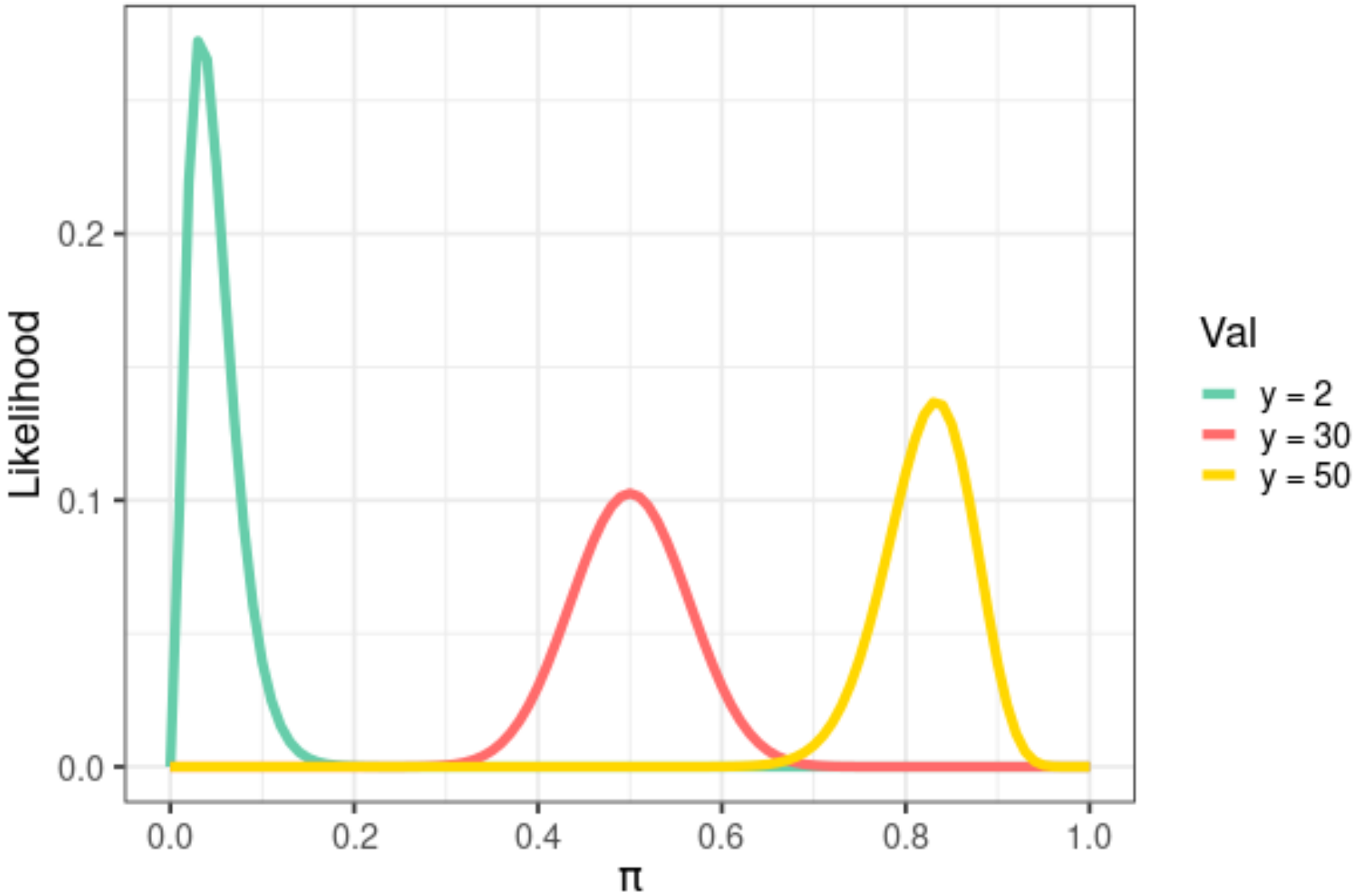
Likelihood Ratios

For $n = 30$



Likelihood Ratios

For $n = 60$



Likelihood Ratios

For $n = 120$