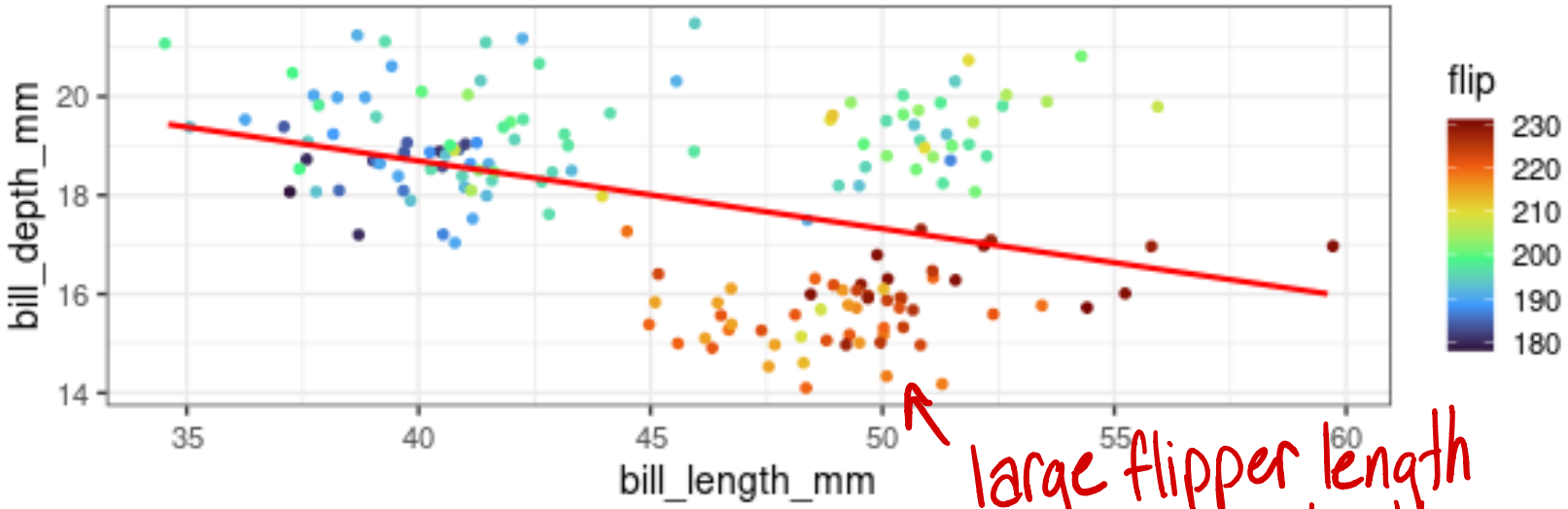
Inference for Multivariate Regression

Annotated!

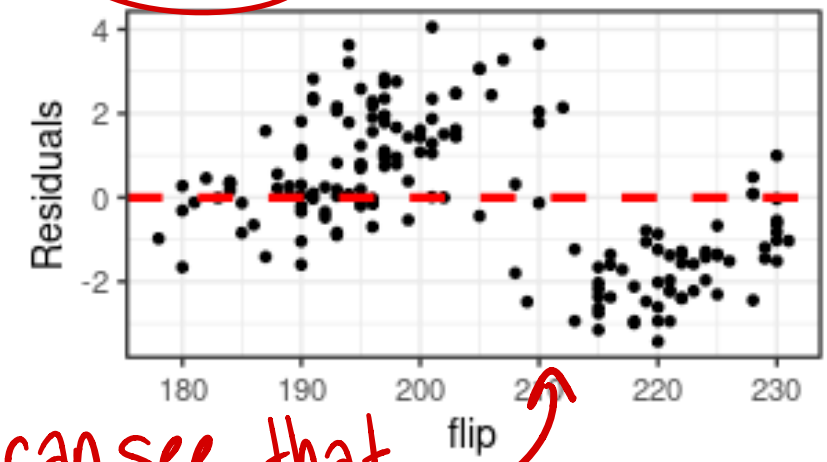
Grinnell College

May 9, 2025

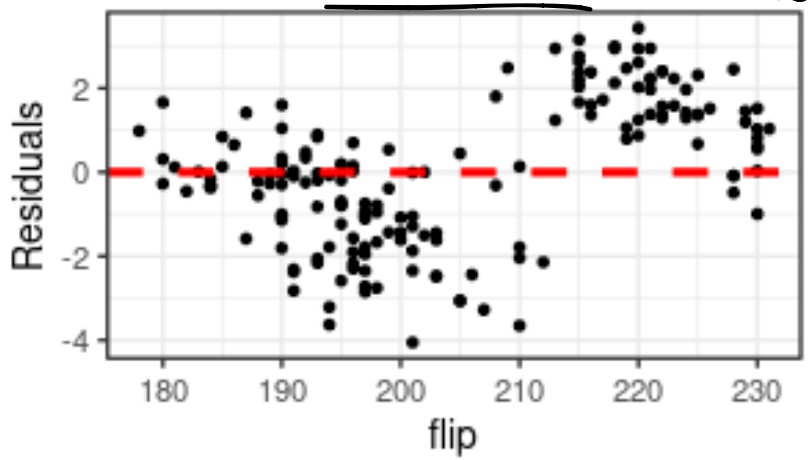


large flipper length associated with negative residuals → less than expected

Plot 1



Plot 2



we can see that relationship here

Cases

1. $y = \beta_0 + X\beta_1$
2. $y = \beta_0 + \mathbb{1}_A\beta_1$
3. $y = \beta_0 + \mathbb{1}_A\beta_1 + X\beta_2$
4. $y = \beta_0 + \mathbb{1}_A\beta_1 + \mathbb{1}_B\beta_2$
5. $y = \beta_0 + X_1\beta_1 + X_2\beta_2$

1. Simple linear, β_1 shows change in y given change in X
2. Simple categorical, reference variable and group means
3. Continuous and categorical, two regression lines with same slope but different intercept
4. Multiple categorical, combined reference variables
5. Multiple continuous, β_1 shows change in y given change in X_1, *assuming everything else held constant*

Single Quantiative

$z_{wt} = \frac{wt - \bar{wt}}{SD}$

Standardized
- Mean = 0
- Sd = 1
- Not normal if original distribution was not normal

→ If you standardize your variables your intercepts become meaningful.

$R^2 = \frac{\text{explained variance}}{\text{explained variance} + \text{unexplaind variance}}$

if it = 1 then all variance is explained by the model.

```
> lm(mpg ~ wt, mtcars) %>% summary()


Coefficients:
             Estimate Std. Error t value            Pr(>|t|)
(Intercept)   37.285      1.878   19.86 < 0.00000000002 ***
wt            -5.344      0.559   -9.56            0.000013 ***


Residual standard error: 3.05 on 30 degrees of freedom
Multiple R-squared:  0.753, Adjusted R-squared:  0.745
F-statistic: 91.4 on 1 and 30 DF,  p-value: 0.000000000129
```

intercept for 2 quantitative: y-value when x=0, in this case that is mpg for a car that weighs nothing.

for every unit increase for weight, we expect mpg to decrease by 5.344

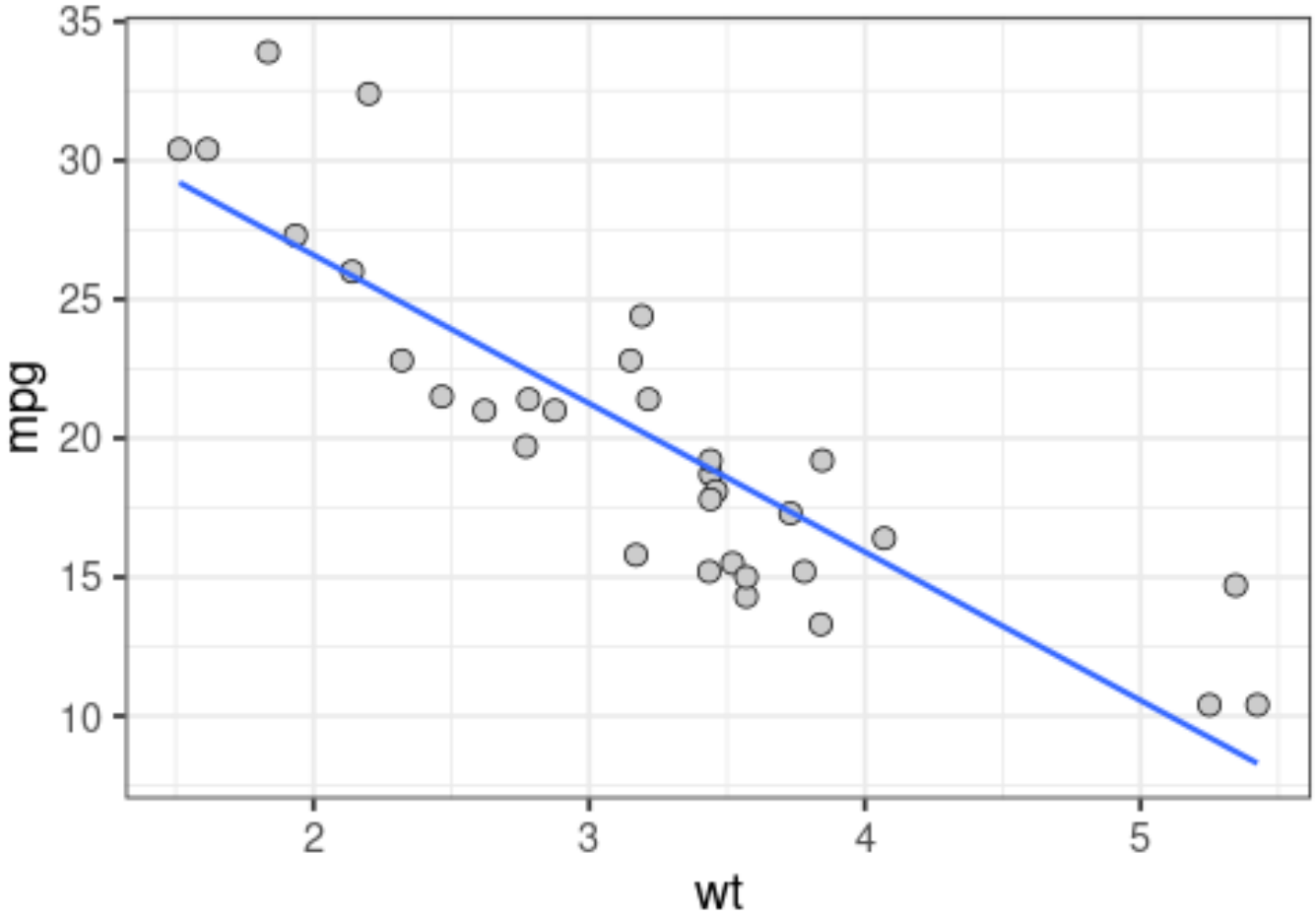
Testing with $\alpha = 0.05$, $p < \alpha$ so we reject.

What proportion of the variance is explained by my model.

With two continuous variables, null hypothesis is that they are not associated, that is $B = 0$. We test, is $B = 0$ with p-value.

Weight and MPG

$$\widehat{\text{mpg}} = 37.285 - 5.34 \times \text{Weight}$$



Single Categorical

```
> lm(mpg ~ cyl, mtcars) %>% summary()

Coefficients:
             Estimate Std. Error t value            Pr(>|t|)
(Intercept)    26.664      0.972   27.44 < 0.00000000002 ***
cyl6           -6.921      1.558   -4.44             0.00012 ***
cyl8          -11.564      1.299   -8.90   0.00000000086 ***
Residual standard error: 3.22 on 29 degrees of freedom
Multiple R-squared:  0.732, Adjusted R-squared:  0.714
F-statistic: 39.7 on 2 and 29 DF,  p-value: 0.00000000498
```

intercept is mean for 4 cyl

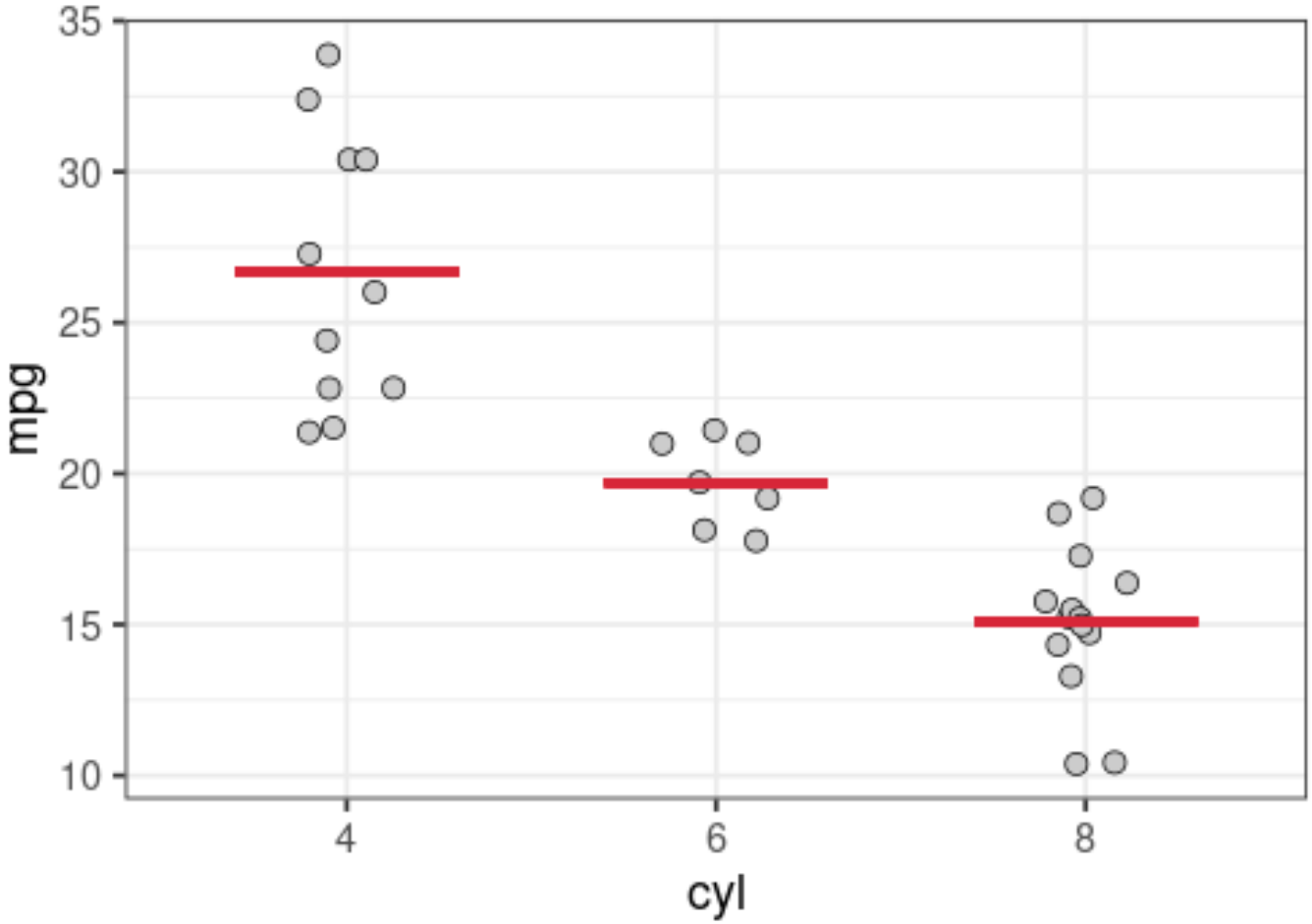
difference between 8 cyl and 4 cyl

Testing $H_0: \mu_8 = \mu_4$

Cylinder and MPG

Regression model with a single categorical variable is equivalent to ANOVA.

$$\widehat{\text{mpg}} = 26.66 - 6.92 \times \mathbb{1}_{\text{6cyl}} - 11.564 \times \mathbb{1}_{\text{8cyl}}$$



← no slopes bc no quantitative variables!

Categorical and Quantitative

```
> lm(mpg ~ wt + cyl, mtcars) %>% summary()

Coefficients:
            Estimate Std. Error t value            Pr(>|t|)
(Intercept)   33.991      1.888   18.01 < 0.00000000002 ***
wt            -3.206      0.754   -4.25             0.00021 ***
cyl6          -4.256      1.386   -3.07             0.00472 **
cyl8          -6.071      1.652   -3.67             0.00100 ***


Residual standard error: 2.56 on 28 degrees of freedom
Multiple R-squared:  0.837, Adjusted R-squared:  0.82
F-statistic: 48.1 on 3 and 28 DF,  p-value: 0.0000000000359
```

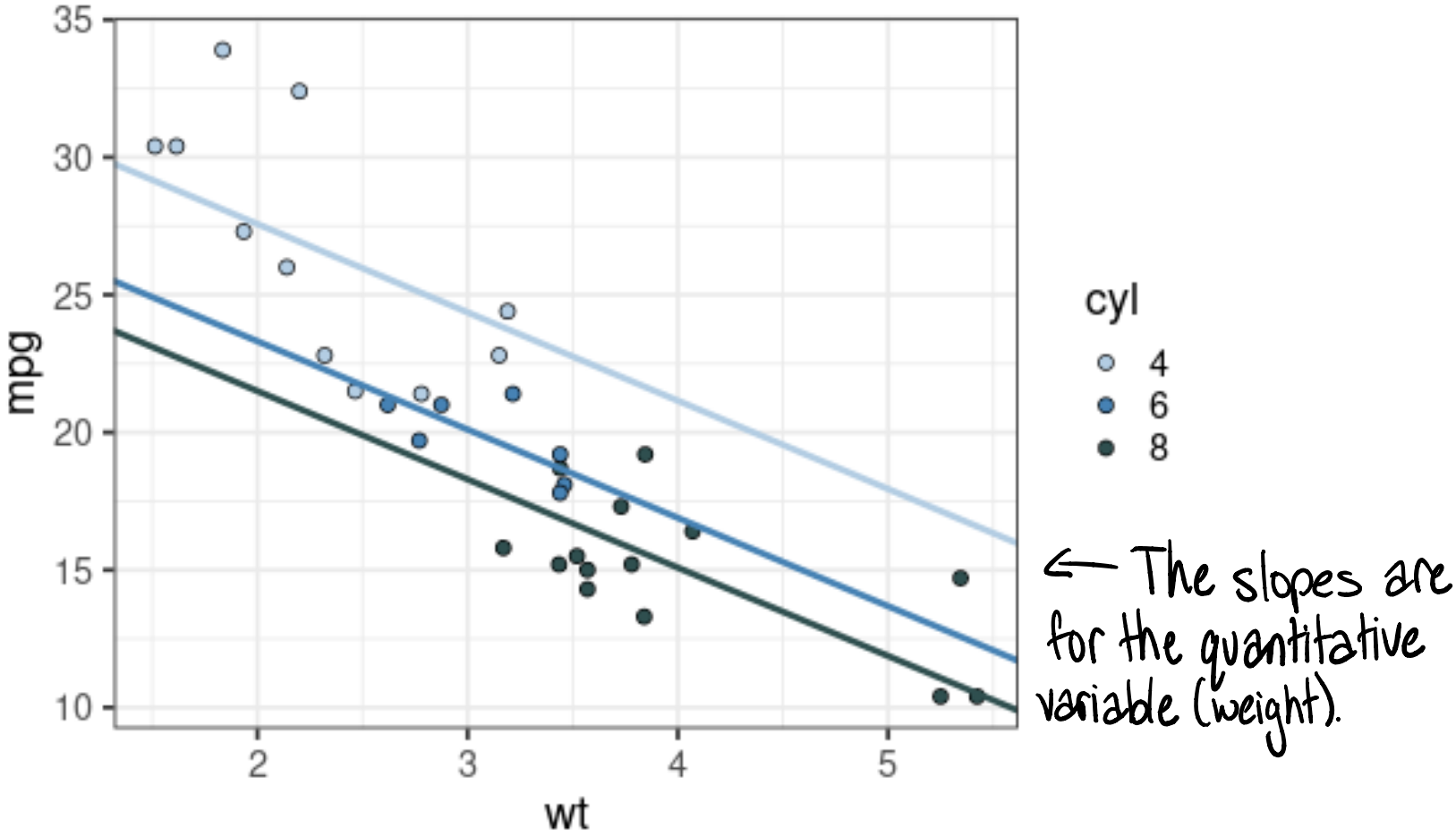
0 weight car that is also 4 cylinders

How much we add to MPG for each unit of weight regardless of cyl's.

difference between 6 cyl and 4 cyl at any weight

Cylinder, weight and MPG

$$\widehat{\text{mpg}} = 33.99 - 3.21 \times \text{weight} - 4.26 \times \mathbb{1}_{\text{6cyl}} - 6.07 \times \mathbb{1}_{\text{8cyl}}$$



← The slopes are for the quantitative variable (weight).

Multiple Categorical

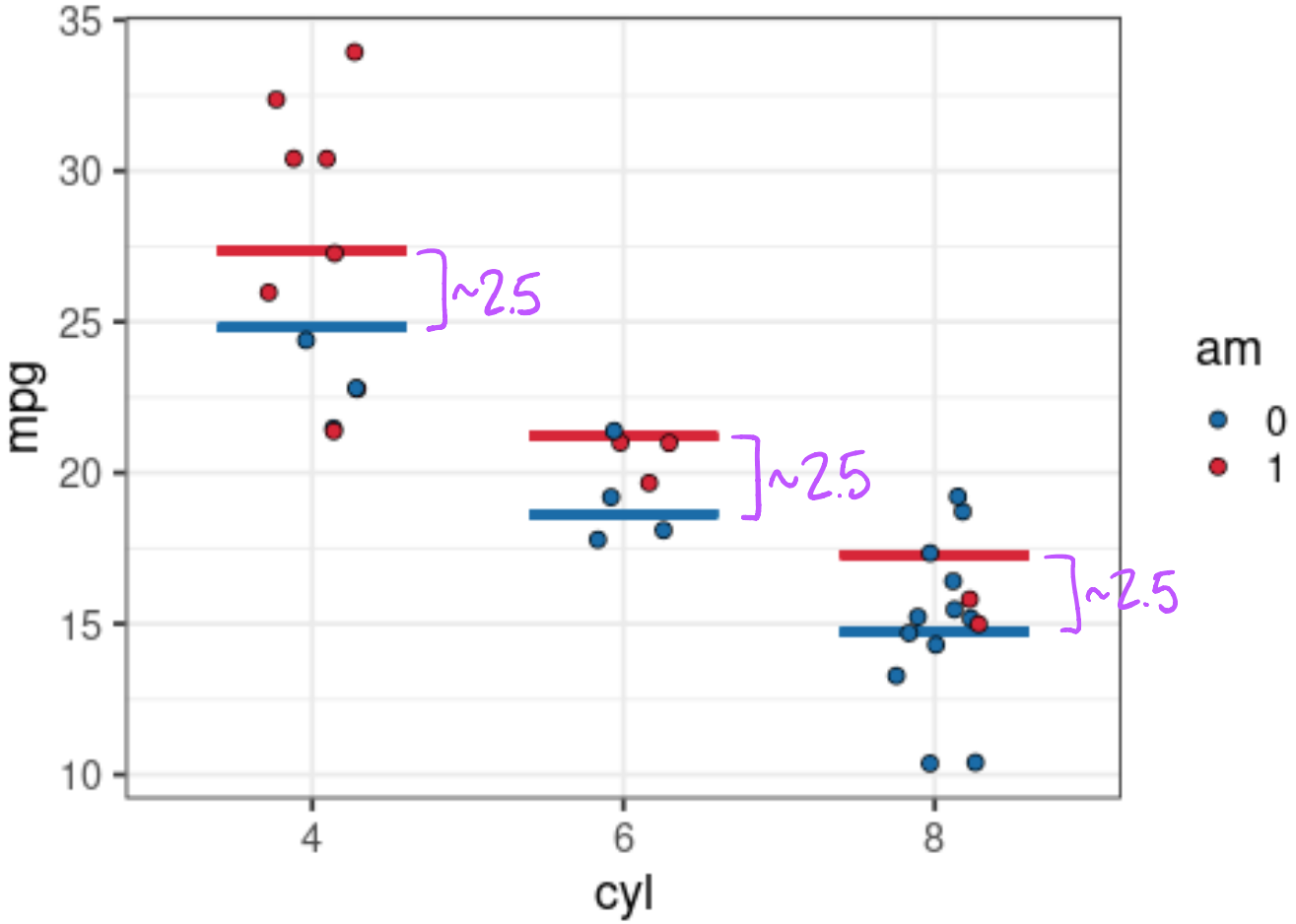
```
> lm(mpg ~ cyl + am, mtcars) %>% summary()

Coefficients:
            Estimate Std. Error t value            Pr(>|t|)
(Intercept)    24.80       1.32   18.75 < 0.00000000002 ***
cyl6           -6.16       1.54   -4.01             0.00041 ***
cyl8          -10.07       1.45   -6.93          0.00000015 ***
am1             2.56       1.30    1.97             0.05846 .

Residual standard error: 3.07 on 28 degrees of freedom
Multiple R-squared:  0.765, Adjusted R-squared:  0.74
F-statistic: 30.4 on 3 and 28 DF,  p-value: 0.00000000596
```

MPG for 4 cyl automatic transmission

difference between auto and manual regardless of cyl

Cylinder, transmission and MPG

$$\widehat{\text{mpg}} = 24.8 - 6.16 \times \mathbb{1}_{\text{6cyl}} - 10.07 \times \mathbb{1}_{\text{8cyl}} + 2.56 \times \mathbb{1}_{\textit{Manual}}$$



Multiple Quantitative

```
> lm(mpg ~ wt + disp, mtcars) %>% summary()

Coefficients:
             Estimate Std. Error t value           Pr(>|t|)
(Intercept) 34.96055    2.16454   16.15  0.000000049 ***
wt          -3.35083    1.16413   -2.8         0.0074 **
disp        -0.01772    0.00919   -1.93        0.0636 .

Residual standard error: 2.92 on 29 degrees of freedom
Multiple R-squared:  0.781, Adjusted R-squared:  0.766
F-statistic: 51.7 on 2 and 29 DF,  p-value: 0.000000000274
```

MPG for a car w/ 0 weight and 0 displacement

wt ← quantitative

disp ↑ quantitative

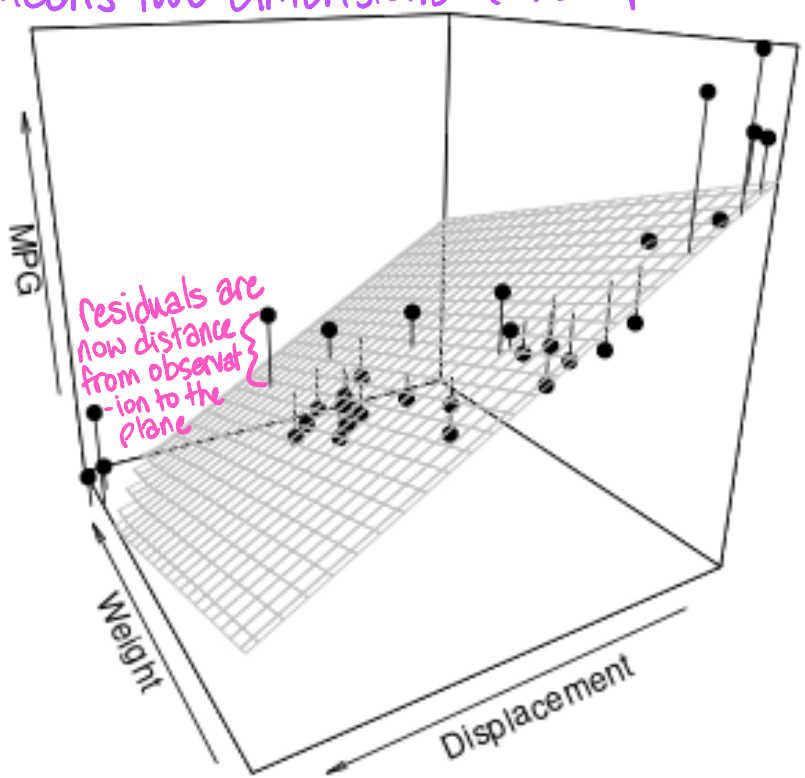
How much you add to MPG for each unit of displacement you ad

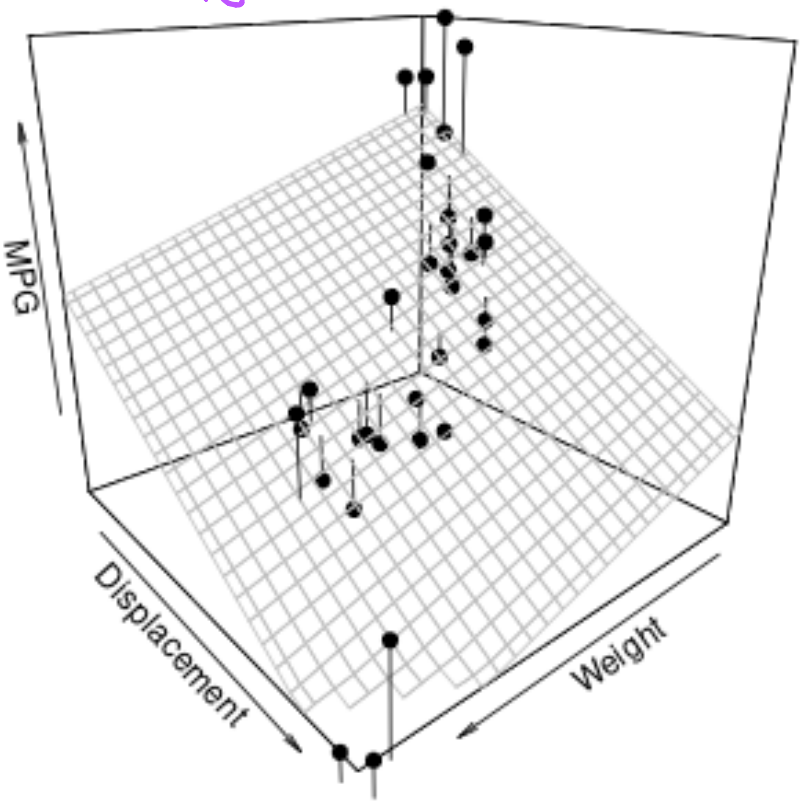
H_0: displacement is not associated w/ mpg. That is slope = 0 / $\beta = 0$.

Cylinder, transmission and MPG

$$\widehat{\text{mpg}} = 34.96 - 3.35 \times \text{weight} - 0.017 \times \text{displacement}$$

The regression line is now a regression plane because two categorical variables means two dimensions! (The dependent variable (mpg) is the third dimension).





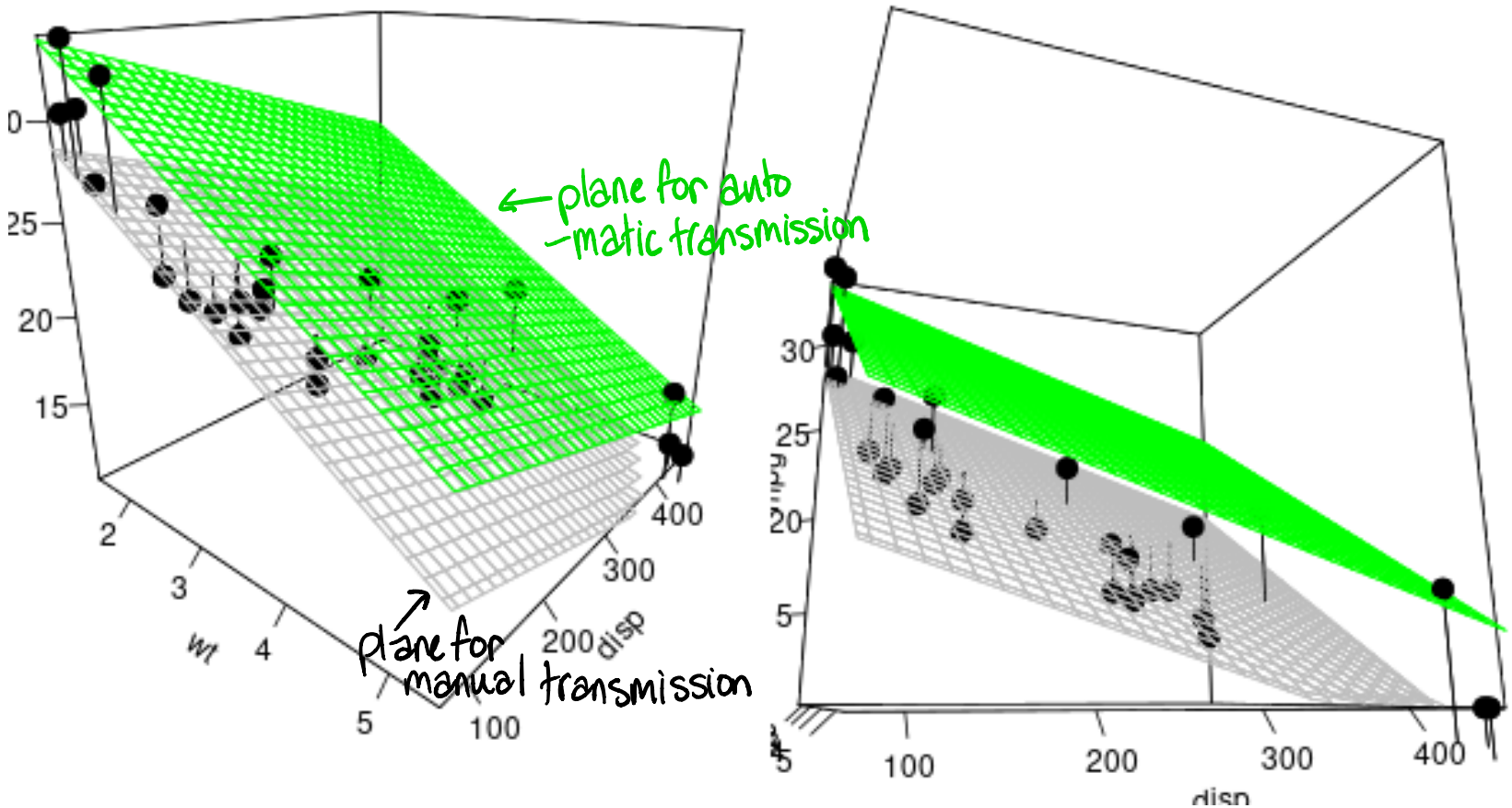
Multiple Quantiative and categorical

```
> lm(mpg ~ wt + disp + am,  mtcars) %>% summary()

Coefficients:
             Estimate Std. Error t value       Pr(>|t|)
(Intercept) 34.67591    3.24061   10.70 0.000000000021 ***
wt          -3.27904    1.32751   -2.47            0.020 *
disp        -0.01780    0.00937   -1.90            0.068 .
am1          0.17772    1.48432    0.12            0.906


Residual standard error: 2.97 on 28 degrees of freedom
Multiple R-squared:  0.781, Adjusted R-squared:  0.758
F-statistic: 33.3 on 3 and 28 DF,  p-value: 0.00000000225
```

MPG for manual transmission car w/ 0 wt and 0 disp

Multiple quantiative with categorical

$$\widehat{\text{mpg}} = 34.67 - 3.27 \times \text{weight} - 0.018 \times \text{displacement} + 0.17 \times \mathbb{1}_{Manual}$$



Key Takeaways

- Quantitative variables represent slopes (changes in X lead to β changes in y)
- Categorical variables represent horizontal shifts
- Any number of categorical or quantitative variables can be added to model
- Lookout for correlated variables
- Always interpret regression coefficients as *everything else being fixed*