Tables Funsheet

Goals

The goals of this funsheet are to investigate properties of independence and association in the context of subsequent coin flips. Specifically, we are trying to ask the question: does data that was collected empirically match our theoretical expectations?

Problem 1

Suppose we conduct an experiment where in each trial, we flip a single coin twice, recording for the first and second toss whether the coin landed on heads or tails. For this first experiment, we performed twenty trials and tabulated the results below. The results of the first flip are given as the row, while the columns represent the outcome of the second flip

	2nd Flip	
	Heads	Tails
Heads	3	5
Tails	6	6

Part A Find the marginal frequencies for the first and second flips. Convert each to a marginal probability.

Part B What is the *marginal probability* of the second coin flip being heads? Express this as $P(H_2) = \ldots$

Part C Find the probability that the second coin lands on heads *given* that the first flip landed on heads. Write this probability of the form $P(H_2|H_1) = \ldots$. Repeat this for $P(T_2|H_1)$ (the probability that the second flip is tails *given* that the first is heads).

Part D Roughly sketch a conditional bar plot, *conditioning on the result for the first flip*

Part E Based on what you found for parts C and D, does it appear as if the coin flips are independent? Specifically, consider the definition of independence, $P(H_2|H_1) = P(H_2)$.

Problem 2

We repeat the experiment above, this time performing $N = 50$ trials

	N = 50	
	Heads	Tails
Heads	13	11
Tails	12	14

Part A Find the marginal frequencies for the first and second flips. Convert each to a marginal probability.

Part B What is the *marginal probability* of the second coin flip being heads? Express this as $P(H_2) = \ldots$

Part C Find the probability that the second coin lands on heads *given* that the first flip landed on heads. Write this probability of the form $P(H_2|H_1) = \ldots$. Repeat this for $P(T_2|H_1)$ (the probability that the second flip is tails *given* that the first is heads).

Part D Roughly sketch a conditional bar plot, *conditioning on the result for the first flip*

Part E Based on what you found for parts C and D, does it appear as if the coin flips are independent? Specifically, consider the definition of independence, $P(H_2|H_1) = P(H_2)$.

Part F Before conducting any trials with coins, we know that the first and second coin flips will be independent. By our definition of independence, however, the marginal probability should be equal to the conditional probability. Provide a few sentences that address the following:

- Why did neither experiment give us exactly what we expected to see under independence?
- Should we be surprised that the first and second experiments gave us different results? Would we expect to see the same number if we reran the first and second experiments with the same number of flips?
- Considering the Law of Large Numbers, why do we expect the second experiment to be closer to our theoretical expectations than the first?
- Will a larger experiment *always* be closer to the theoretical expectation? Why or why not?