Hypothesis Testing: Same Data, Different Decisions?

Use your **critical value sheet** and a calculator. Assume independent random samples from normal populations. All tests are **two-sided**: $H_0 : \mu = \mu_0$ versus $H_A : \mu \neq \mu_0$.

$$SE = \frac{s}{\sqrt{n}}, \qquad t = \frac{\overline{x} - \mu_0}{SE}, \qquad \text{CI: } \overline{x} \pm C \times SE, \qquad df = n - 1.$$

1. Same sample, different confidence levels

A sample gives $\overline{x} = 105$, $s = 10$, and $n = 10$. Test $H_0 : \mu = 100$ at **80% and 90% confidence**.

(a) Find SE, df, and t. Complete the table. Do the CI method and the $|t|$ versus C method give the same decision?

Confidence	C	Confidence interval	$\lvert t\rvert < C$?	Decision about H_0
80%				
90%				

(b) Suppose H_0 is actually true. Why is the larger confidence level better for avoiding an incorrect rejection? If we repeated this process many times, about what percentage of samples would lead us to reject a true H_0 at each confidence level?

(c) What does it mean that we reject at 80% confidence but fail to reject at 90% confidence? What changed, and what stayed the same? Does failing to reject establish that $\mu = 100$?

(d) Now suppose a sample has **twice as many observations** ($n = 20$), but the same $\overline{x} = 105$ and $s = 10$. At **90% confidence**, find the new SE, C, and t, and make a decision. Why does this decision differ from the original sample's 90% decision? Address both SE and C.

2. Same sample, different null hypotheses

Use $\overline{x} = 105$, $s = 10$, and $n = 20$. Consider two possible null hypotheses, each with a two-sided alternative:

$$H_{0,A} : \mu = 100, \qquad H_{0,B} : \mu = 104.$$

(a) At **90% confidence**, calculate t for each hypothesis and compare $|t|$ to C. State the decision for each. Why are the t statistics different even though the data are identical?

(b) Which hypothesized mean is more consistent with these data? Explain using distance from $\overline{x}$ in **standard errors**. Does that establish that this hypothesized mean is the true mean?

Bonus: Same null hypothesis, different samples

Two independent samples are used to test $H_0 : \mu = 100$, each against $H_A : \mu \neq 100$.

Sample	$\overline{x}$	s	n
A	105	10	20
B	107	20	20

(a) Calculate both t statistics. Which sample is more consistent with H_0? Why can the sample mean farther from 100 give a smaller $|t|$?

(b) If the sample sizes were different, what would you need to recalculate? Would both statistics still be compared with the same t distribution? Describe how you might compare how unusual the two results are under H_0.