Bounding p-values: A Tale of Two Tails

0. Confidence levels and error rates

For a test using each confidence level below, give the associated **Type I error rate**, α.

Confidence level	80%	90%	95%	99%
α				

If the null hypothesis is true, what does α tell us about the decisions we would make over many repeated samples?

1. Finding bounds without calculating an exact p-value

In trying to determine the time waiting in line at D Hall when entering on noon on a MWF, one student posited that the average waiting time was 13 minutes. A sample of 27 waiting times was collected, with a mean waiting time of $\overline{x} = 14.7$ minutes and with $s = 4.7$ minutes. Test $H_0 : \mu = 13$ against $H_A : \mu \neq 13$.

(a) Calculate $SE = s/\sqrt{n}$, $df = n - 1$, and $t = (\overline{x} - 13)/SE$.

(b) Complete the table using your critical value sheet. For each row, does the comparison of $|t|$ with C tell you that $p < \alpha$ or $p > \alpha$?

Confidence	α	C	$\lvert t\rvert > C$?	$p < \alpha$ or p $> \alpha$?	Reject or fail to reject?
90%					
95%					

(c) Combine your two comparisons to put bounds on p:

_____ $< p <$ _____.

Explain why being farther into the tails gives a smaller p-value.

2. Which sample gives stronger evidence?

Two students independently sample turtles of the same species at Arbor Lake to estimate the mean shell length. Student A finds **13 turtles**; Student B finds **22 turtles**. Both assess a claim that the population mean shell length is **22.4 cm**, testing $H_0 : \mu = 22.4$ against $H_A : \mu \neq 22.4$. Their sample summaries are below; lengths are in centimeters.

Sample	$\overline{x}$	s	n
A	25.3	3.5	13
B	25.0	4.2	22

(a) Before calculating: which sample do you predict will give stronger evidence against H_0? Give a reason. You may revise your prediction later.

(b) Calculate SE, df, and t for each sample. Find the 95% and 99% critical values for each sample's degrees of freedom, and use them to bound its p-value. You do not need an exact p-value.

Sample	SE	df	t	95% C	99% C	Bounds on p
A						
B						

(c) Which sample has the larger $|t|$? Which gives stronger evidence against H_0? Explain why comparing the two $|t|$ values alone can be misleading. Refer to the degrees of freedom and your bounds on p.

(d) State the decision for each sample at each Type I error rate below. How can you make these decisions without knowing the exact p-values?

Sample	$\alpha = 0.05$	$\alpha = 0.01$
A		
B		