Probability Worksheet

Day 3

Rules

For each of these problem, please use notation that we have adopted in class, i.e., events A or B, probabilities $P(A)$, expressions $P(A|B)$ or $P(A \text{ or } B)$, etc., in addition to solving them numerically.

Sum of Condiditional Probabilities

Bayes Theorem States that we find the probability of A given B with the following expression:

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

We have from class that the sum of conditional probabilities is equal to 1, i.e.,

$$1 = P(A|B) + P(A^C|B)$$

Using the various identities that we have covered this week, manipulate the sum of conditional probabilities above to show that

$$P(B) = P(B|A) \times P(A) + P(B|A^C) \times P(A^C)$$

$$1 = P(A|B) + P(A^c|B)$$

$$1 = \frac{P(A \cap B)}{P(B)} + \frac{P(A^c \cap B)}{P(B)}$$

$$P(B) = P(A \cap B) + P(A^c \cap B)$$

$$P(B) = P(B|A)P(A) + P(B|A^c)P(A^c)$$

A = Academic Event + = garage full

S = Sport Event − = garage not full

N = None

$P(A) = 0.35$ $P(+|A) = 0.25$

$P(S) = 0.2$ $P(+|S) = 0.7$

$P(N) = 0.45$ $P(+|N) = 0.05$

Bayes Theorem

Jose visits campus every Thursday evening. However, some days the parking garage is full, often due to college events. There are academic events on 35% of evenings, sporting events on 20% of evenings, and no events on 45% of evenings. When there is an academic event, the garage fills up about 25% of the time, and it fills up 70% of evenings with sporting events. On evenings when there are no events, it only fills up about 5% of the time.

Question 1 If Jose comes to campus and finds the garage is full, what is the probability that there is a sporting event?

$$P(S|+) = \frac{P(+|S)P(S)}{P(+)}$$

$$= \frac{(0.7)(0.2)}{0.25} = \boxed{0.56}$$

$$P(+) = P(+|A)P(A) + P(+|S)P(S) + P(+|N)P(N)$$

$$= 0.25$$

Question 2 Using the information provided, verify that the probability that there is an academic event, given that the garage is full, is equal to 0.35.

$$P(A|+) = \frac{P(+|A)P(A)}{P(+)}$$

$$= \frac{(0.25)(0.35)}{0.25} = 0.35$$

Question 3 Using results from the previous problems, what is the probability that there is no event, given that the lot is full?

$$P(+|N) = 1 - P(+|A) - P(+|S)$$

$$= 0.09$$

Question 4 (This question is also on the homework) A genetic test is used to determine if people have a predisposition for thrombosis, which is the formation of a blood clot inside a blood vessel that obstructs the flow of blood through the circulatory system. It is believed that 3% of people actually have this predisposition. The genetic test is 99% accurate if a person actually has the predisposition, meaning that the probability of a positive test result when a person actually has the predisposition is 0.99. The test is 98% accurate if a person does not have the predisposition. What is the probability that a randomly selected person who tests positive for the predisposition by the test actually has the predisposition?

$P(T) = 0.03$ $\quad P(T^c) = 0.97$

$P(+|T) = 0.99$ $\quad P(-|T^c) = 0.98$

$$P(T|+) = \frac{P(+|T)P(T)}{P(+)}$$

$$= \frac{P(+|T)P(T)}{P(+|T)P(T) + P(+|T^c)P(T^c)}$$

$$= \frac{(0.99)(0.03)}{(0.99)(0.03) + (0.02)(0.97)}$$

$$= 0.6049$$