Probability Worksheet

Day 2

Rules

For each of these problem, please use notation that we have adopted in class, i.e., events A or B, probabilities $P(A)$, expressions $P(A|B)$ or $P(A \text{ or } B)$, etc., in addition to solving them numerically.

Independence

Question 1 It is estimated that 9% of people are left handed. We randomly sample five people from our population. Based on these five, answer the following:

- What is the probability that all five people are right-handed?
- What is the probability that all five people are left-handed?
- What is the probability that that not all five people are right-handed (i.e., probability that *at least one* is left-handed)?

- $(0.91)^5$
- $(0.09)^5$
- $1 - (0.91)^5$

Question 2 Further assume that sex and handedness are independent, i.e.,

$$P(\text{right-handed and male}) = P(\text{right-handed}) \times P(\text{Male}).$$

In a population with equal proportions of men and women, answer the following:

- What is the probability that the first person is male and right-handed?
- What is the probability the the first two people are male and right-handed
- What is the probability that the third person is female and left-handed?
- What is the probability that the first two people are male and right-handed and the third person is female and left-handed?

- $(0.91)(0.5)$
- $[(0.91)(0.5)]^2$
- $\cancel{P}(0.09)(0.5)$
- $[(0.91)(0.5)]^2 \times (0.09)(0.5)$

Question 3 In a standard 52 card deck, assume we draw one card at random. Is the event that a card is heart independent of the card being an ace? Make your argument using the **multiplication rule**

$P(\text{Ace}) = \cancel{}\ 4/52 \quad P(\text{Heart}) = 13/52$

$P(\text{Ace and } \heartsuit) = (4/52)(13/52) = 1/52$

Conditional Probability

Below is a table of 6,224 individuals from the year 1721 who were exposed to smallpox in Boston. Doctors at the time believed that exposing a person to the disease in a controlled form (inoculating them) could reduce the likelihood of death.

	inoculated		
	yes	no	Total
lived	238	5136	5374
died	6	844	850
Total	244	5980	6224

Here, the same table is reproduced, providing joint and marginal probabilities

	inoculated		
	yes	no	Total
lived	0.0382	0.8252	0.8634
died	0.0010	0.1356	0.1366
Total	0.0392	0.9608	1.0000

Question 1 Write in formal notation the probability that a randomly selected person who was not inoculated died from smallpox and compute the probability

$$P(\text{died} \mid \text{no inoculation}) = \frac{844}{5980} \approx 0.14$$

Question 2 Determine the probability that an inoculated person died from smallpox. How does this compare with what you found in Question 1?

$$P(\text{died} \mid \text{inoculated}) = \frac{6}{244} = 0.02$$

Question 3 Based on your results from Q1 and Q2, does it appear as if inoculation is effective at reducing the risk of smallpox?

Yes, probability of dying much higher in no inoculation group

Question 4 Let X and Y represent the outcomes of rolling two dice. Answer the following:

- What is the probability that the first die, X, is equal to 1?
- What is the probability that both X and Y are equal to 1?
- Use the formula for conditional probability to compute $P(Y = 1|X = 1)$
- What is $P(Y = 1)$? Is this different that what we found in the last part? Explain.

$P(X=1) = 1/6$

$P(X=1 \text{ and } Y=1) = 1/36$

$$P(Y=1/X=1) = \frac{P(Y=1 \text{ and } X=1)}{P(X=1)} = \frac{(1/36)}{(1/6)} = 1/6$$

$P(Y=1) = P(Y=1/X=1)$ b/c independent

Question 5 Suppose that 80% of people like peanut butter (obviously), 89% like jelly, and 78% like both. Given that a randomly sampled person likes peanut butter, what is the probability that they also like jelly?

$P(PB) = 0.8 \quad P(J) = 0.89 \quad P(PB \text{ and } J) = 0.78$

$$P(J|PB) = \frac{P(PB \text{ and } J)}{P(PB)} = \frac{0.78}{0.8} = 0.97$$

Question 6 Bob is watching a roulette table in a casino and notices that the last five outcomes landed on black. He figures that the probability of landing on black six times in a row is very small (1/64), so he places a bet on red. What is wrong with this logic?

The outcome of roll 6 independent from first 5

Birthday Problem What is the probability that *at least* two people in a room share a birthday? For this problem, we can assume:

- There are 25 individuals in the classroom
- Nobody has a birthday on Feb 29
- Any individual person is equally likely to be born on any day of the year, i.e., the probability of being born on each day is the same
- Hint – what does this produce: `prod(1:4)`?

Work with your group to come up with a solution to this problem. Try using the notation we have covered today and Monday to frame this problem in probabilistic statements. You are welcome to use R for this problem

A = At least 2 share bday

A^c = Nobody share bday

$$P(A^c) = \left(\frac{365}{365}\right)\left(\frac{364}{365}\right)\left(\frac{363}{365}\right) \cdots \left(\frac{341}{365}\right)$$

prob that person 1 has bday

Probability that person 2 don't share w/ 1

Probability that 3 doesn't share w/ 1 or 2

$$\Rightarrow P(A) = 1 - P(A^c)$$

$$= 1 - \left(\frac{365}{365}\right)\left(\frac{364}{365}\right) \cdots \left(\frac{341}{365}\right)$$

$$= 1 - \left(\frac{1}{365}\right)^{25} \times \text{prod}(341:365)$$

$$= 0.5687$$