Probability Worksheet

Day 1

Introduction

These initial problems will help get us oriented in a context that is more familiar. In each, we will be referring to a standard "die" (singular of dice) that has 6 faces, each with an equal chance of being rolled

Part A What is the chance of getting a 1 when rolling a dice?

$$P(1) = \frac{1}{6}$$

Part B What is the chance of rolling a 1 or a 2 in the next roll?

$$P(1 \text{ or } 2) = \frac{2}{6} = \frac{1}{3}$$

Part C What is the chance of rolling a 1,2,3,4,5, or 6?

$$P(1, 2, 3, 4, 5, 6) = \frac{6}{6} = 1$$

Part E What is the chance of *not* rolling a 2?

$$P(1, 3, 4, 5, 6) = \frac{5}{6} \qquad \text{or} \qquad 1 - P(2) = 1 - \frac{1}{6} = \frac{5}{6}$$

Problem 1 (Addition Rule)

Question 1 Here, we concern ourselves with 10,000 individuals who either rent their home (3858), have a mortgage on their home (4789), or own it outright (1353).

- What proportion of individuals have either a mortgage or own it outright?
- If we select one person out of this 10,000 at random, what is the probability that this person either owns their own or has a mortgage?

$$\text{Proportion mortgage or own} = \frac{4789 + 1353}{10000} = 0.6142$$

In this case, the proportion is our estimate of the probability, so the probability is 0.6142

Question 2 Consider rolling a dice where we define three different events:

$$A = \{1, 2\}, \quad B = \{4, 6\}, \quad D = \{2, 3\}$$

- What is the probability of event A?
- Are events B and D disjoint? Confirm the addition rule by finding the probability that either B or D occurs.

$$P(A) = \frac{2}{6}$$

A and B are disjoint.

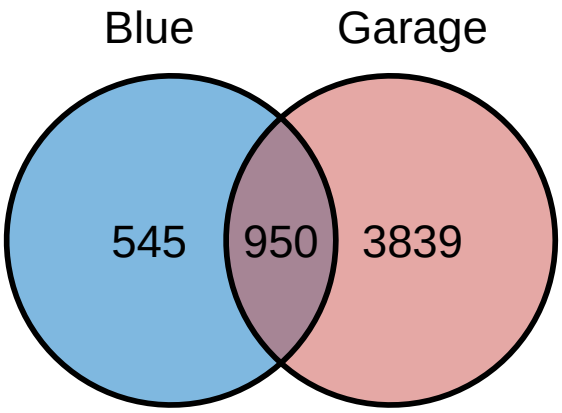
$$P(A) = \frac{2}{6}, \qquad P(B) = \frac{2}{6}, \qquad P(A \text{ or } B) = P(A) + P(B) = \frac{4}{6}$$

Problem 2 (General Addition Rule)

Question 1 If events A and B are disjoint, explain why this implies that $P(A \text{ and } B) = 0$. Verify that the General Addition Rule simplifies to the Addition Rule when A and B are disjoint.

If A and B are disjoint, then there are no outcomes that exist in both A and B. As such, the probability of their intersection is 0.

Question 2 In a sample of 10,000 homes, 1495 homes were painted blue, 4789 had a garage, and 950 homes had both of these properties. Create a Venn diagram illustrating this problem.



Problem 3 Using the information from Question 2, what is the probability that a home selected at random had a garage but was not painted blue?

$$\frac{3839}{10000}$$

Problem 3 (Compliments)

Question 1 For a single dice roll, let $D = \{2, 3\}$. What is D^C? Find $P(D)$ and $P(D^C)$?

$$D^C = \{1, 4, 5, 6\}, \qquad P(D) = \frac{2}{6}, \qquad P(D^C) = \frac{4}{6}$$

Below is a table showing the probability of finding a sum after rolling two dice

Dice Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Question 2 Let A represent the event in which we roll two dice and their total is less than 12. What does A^C represent?

If A is the sum is less than 12 then A^C is the event we roll a 12

Question 3 Find the following probabilities from rolling two dice:

1. The sum of the dice is *not* 6
2. The sum is *at least* 4
3. The sum is not more than 10

1. $A = \{1, 2, 3, 4, 5\}$ so $A^C = 6$, $P(A) = 1 - P(A^C) = 1 - \frac{5}{36} = \frac{31}{36}$
2. $A^C = \{2, 3\}$ so $P(A) = 1 - P(A^C) = 1 - \frac{3}{36} = \frac{33}{36}$
3. $A^C = \{10, 11, 12\}$ so $P(A) - 1 - P(A^C) = \frac{33}{36}$