

# Data Visualization

**Explanatory variable** – suspected cause (independent variable)

**Response variable** – suspected effect (dependent variable)

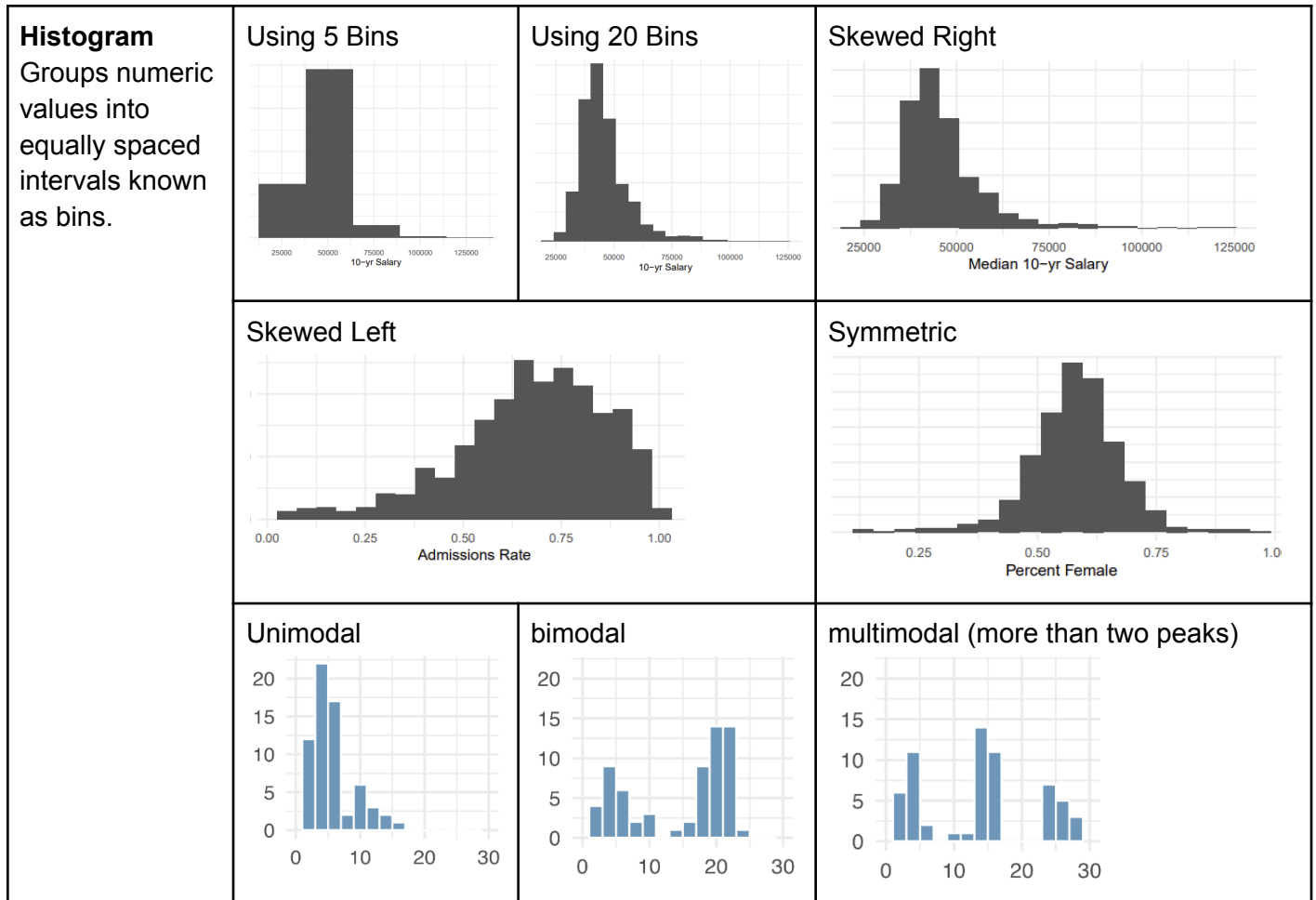
**Univariate graph** – show the distribution of a single variable.

**Bivariate graph** – show the relationship between two variables.

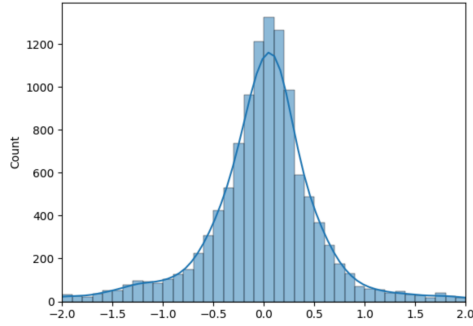
## One Categorical



## One Quantitative



**Density plot**  
a smoothed out  
histogram

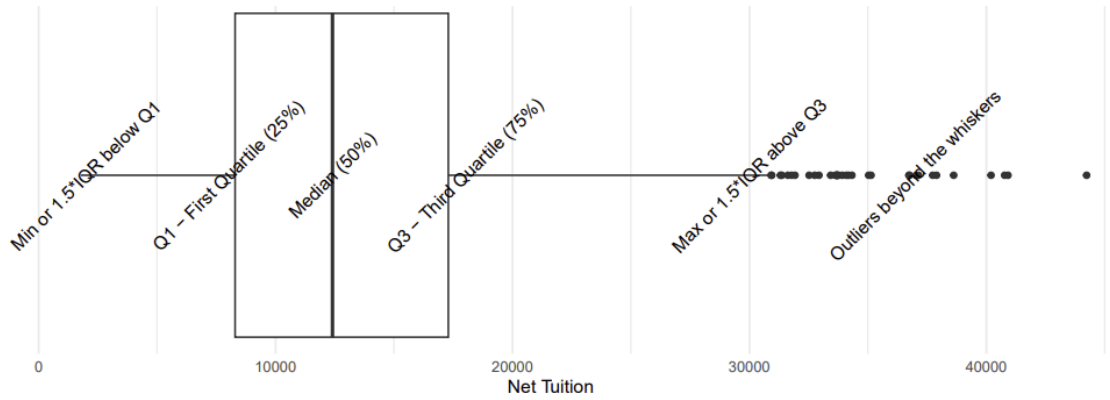


**Box Plot**

**percentile  $\alpha$**  – a number such that  $\alpha\%$  of our (quantitative) observations fall below this number.

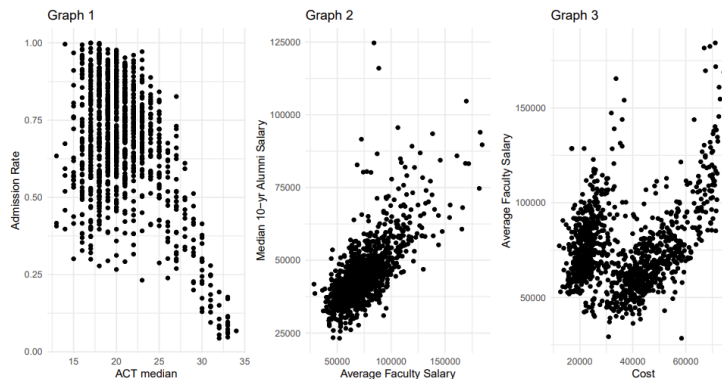
$$\text{Interquartile range} = Q_3 - Q_1$$

$$\begin{aligned} \text{Outlier} &= 1.5 \times IQR + Q_3 \\ &= Q_1 - 1.5 \times IQR \end{aligned}$$

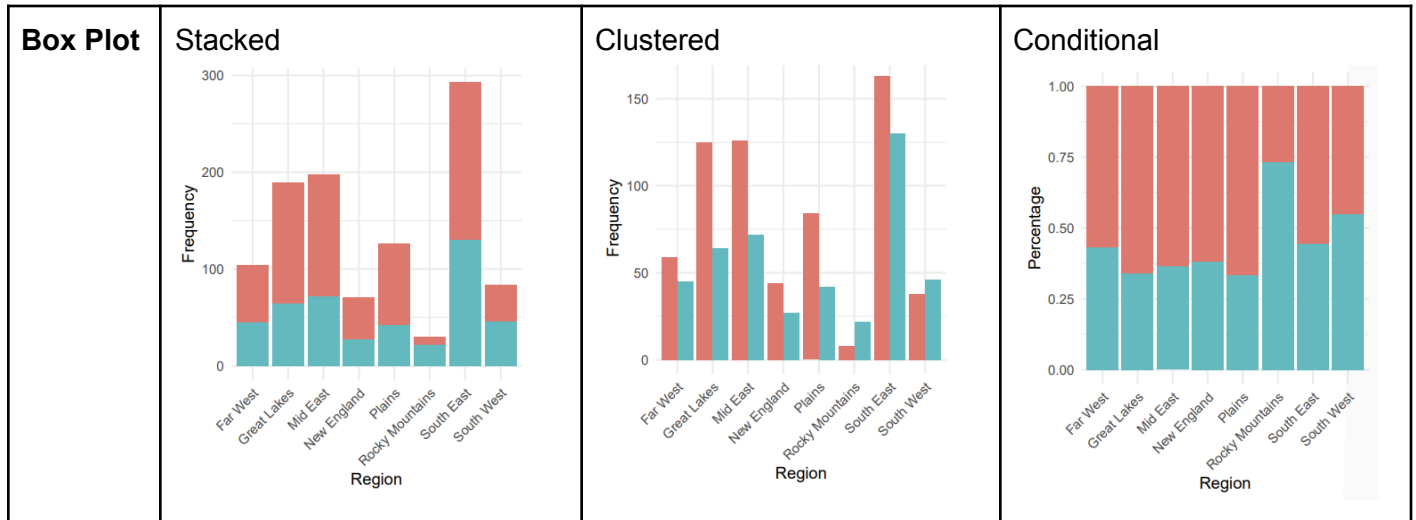


## Two Quantitative

**Scatter Plot**



# Two Categorical



## One quantitative variable:

1. Shape – is the distribution symmetric, skewed, bell-shaped, bimodal
2. Center – where are the data centered (mean and median)
3. Variability – How spread out are the data (range)
4. Unusual Points – outliers or excessive zeros

## Two quantitative Variables:

1. Form – what type of trend or pattern exist(linear, non-linear, exponential, etc)
2. Strength – how closely do the data adhere to a trend or pattern (strong, moderate, weak)
3. Direction – how the values of one variable relate to the values of another variable(positive, negative)
4. Unusual Points – outliers or excessive zeros

# Numerical Summaries

**robust statistic** – statistics that tends to not be influenced by outliers

## – Measures of centrality

<b>Mean (<math>\bar{x}</math>)</b>	the arithmetic average of a variable ( <b>not robust</b> )	$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$
<b>Median</b>	the middle value of the data if arranged from smallest to largest. ( <b>robust</b> )	

## – Measures of Spread

<b>Standard deviation(<math>\sigma</math>)</b>	the average deviation(distance) of individual observations from the mean value. ( <b>Not robust</b> )	$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$
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		n = number of observation
<b>Variance(<math>\sigma^2</math>)</b>	measure that quantifies how much a set of numbers deviate from their mean.	$variance = \sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2$
<b>Range</b>	the difference between smallest and largest values	
<b>Interquartile Range</b>	the difference between the 75th quartile and the 25th quartile. ( <b>Robust</b> )	$Interquartile\ range = Q_3 - Q_1$

## Tables and Odds

### – Tables

**Contingency table** – two way table in which both categorical variables have a binary response

	Event	Non-Event
Exposure	A	B
No Exposure	C	D

**Exposure** – presence of a factor that is being studied to determine its effect on a particular outcome

**No Exposure** – absence of a factor that is being studied to determine its effect on a particular outcome

**Event** – the occurrence of the outcome of interest

**Non-Event** – outcome did not occur

### – Odds

$$Probability = \frac{\text{number of success}}{\text{total number}}$$

$$Odds = \text{number of success} : \text{number of fail} = \frac{\text{number of success}}{\text{total number} - \text{number of success}}$$

$$Odds\ Ratio = \frac{\text{Odds of event in the exposed group}}{\text{Odds of event in the unexposed group}}$$

<b>OR = 1</b>	No association between the exposure and the outcome.
<b>OR &gt; 1</b>	Positive association – the exposure increases the likelihood of the event.  Example: OR = 2, the event is twice as likely to occur in the exposed group as in the unexposed group.
<b>OR &lt; 1</b>	Negative association –the exposure decreases the likelihood of the event.

Example:  
OR = 0.5, the event is half as likely to occur in the exposed group as in the unexposed group.

## Z-Score

**z-scores** describes a value's relationship to the mean of a group of values.

The Z score of an observation is defined as the number of standard deviations it falls above or below the mean.

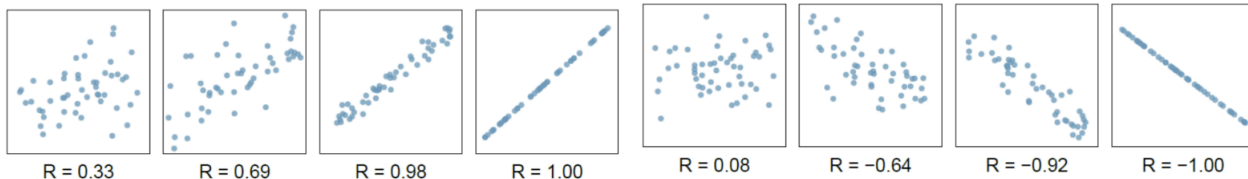
Z score of  $X_1$  is  $Z_1$  and Z score of  $X_2$  is  $Z_2$ . If  $|Z_1| > |Z_2|$ , then  $X_1$  is more unusual than  $X_2$ .

Examples	Equation
Observation is one standard deviation above the mean, Z score = 1.	$Z_i = \frac{x_i - \bar{x}}{S_x}$ $x_i$ = one quantitative variable $\bar{x}$ = mean value of the variable $S_x$ = standard deviations above/below the average
Observation is 1.5 standard deviations below the mean, Z score = -1.5.	

## Regression

### - Correlation

Correlation is stronger if r is closer to 1 or -1, weaker if r is closer to 0.



<b>Pearson's correlation coefficient</b> measures the strength of linear association between two quantitative variables	$r = \frac{1}{n-1} \sum_{i=1}^n \left( \frac{x_i - \bar{x}}{s_x} \right) \left( \frac{y_i - \bar{y}}{s_y} \right)$ $= \frac{1}{n-1} \sum_{i=1}^n (z_{x_i})(z_{y_i})$
<b>Spearman's rank correlation</b>	

measures the strength of monotonic (non-linear) association between two quantitative variables	
<b>Ecological correlations</b> compare variables for data that have been aggregated at an ecological level. A correlation between two variables that are group means	<b>Ecological fallacy</b> – Ecological fallacies occur when we try to draw conclusions about individuals based on data collected at the group level.

## – Regression line

Quantitative Regression	Binary Categorical Regression
$y = \beta_0 + x\beta_1$  $\beta_0$ = intercept $\beta_1$ = slope  $\hat{y}$	$reference\ type = \beta_0 + 1_1\beta_1 + 0_2\beta_2$  $\beta_1, \beta_2$ = Coefficient $1_1, 0_2$ = Indicator Variables $\beta_0$ = Average of the reference type $\beta_1 + \beta_0$ = average for one indicator variable $\beta_2 + \beta_0$ = average for the other indicator variable

## – Coefficient of determination $R^2$

<b>Total sum of squares (SST)</b>	distance between the mean of the observed and actual	$SST = \sum_{i=1}^n (y_i - \bar{y})^2$
<b>Residual Sum of squares (SSR)</b>	distance between the expected and actual	$SSR = \sum_{i=1}^n (y_i - \hat{y}_i)^2$
<b>Coefficient of determination</b>	proportion of the variance for a response variable that is explained by one or more explanatory variables	$R^2 = 1 - \frac{SSR}{SST} = r^2$

$R^2$  range from 0 to 1

- $R^2 = 1$  → regression model perfectly explains all the variability of the response variable
- $R^2 = 0$  → regression model explains none of the variability of the explanatory variables

### Example:

$R^2 = 0.8$  → 80% of the variability in the response variable is explained by the explanatory variables, while the remaining 20% is unexplained and may be due to other factors or random noise.